CMPS 101
Summer 2008
Homework Assignment 7

1. (1 Point) p.136: 6.4-2
   Argue the correctness of Heapsort using the following invariant:
   > At the start of each iteration of the for loop on lines 2-5, the subarray $A[1\cdots i]$ is a max-heap containing the i smallest elements of $A[1\cdots n]$, and the subarray $A[(i+1)\cdots n]$ contains the $n-i$ largest elements of $A[1\cdots n]$ in increasing order.

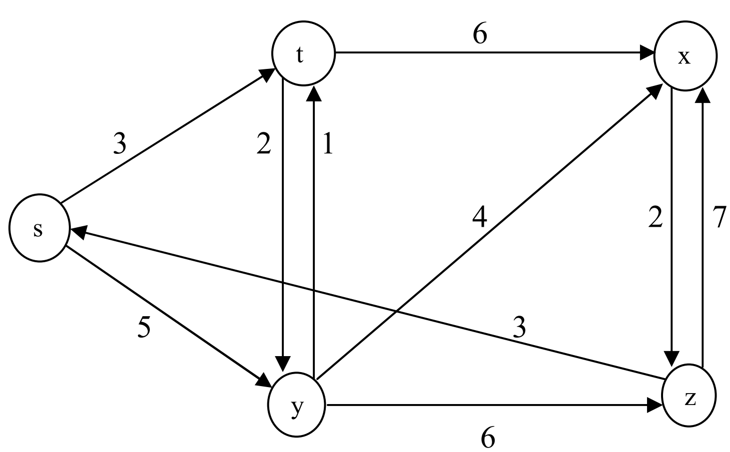
   We reproduce the Heapsort pseudo-code below for reference:

   <u>HeapSort(A)</u>
   1. BuildMaxHeap(A)
   2. for $i \leftarrow$ length[A] down to 2
   3. $A[1] \leftrightarrow A[i]$
   4.   heap-size[A] $\leftarrow$ (heap-size[A] $-$ 1)
   5.   MaxHeapify(A, 1)

2. (10 Points)
   Let $x \in V(G)$ and suppose that after INITIALIZE-SINGLE-SOURCE(G, s) is executed, some sequence of calls to Relax( ) causes $d[x]$ to be set to a finite value. Then G contains an s-x path of weight $d[x]$. (Hint: Use induction on the length of the Relaxation sequence, and recall that this result was proved in class.)

3. (1 Point) p.591: 24.1-3
   Given a weighted, directed graph $G=(V,E)$ with no negative-weight cycles, let m be the maximum over all pairs of vertices $u,v \in V$ of the minimum number of edges in a shortest path from u to v. (Here, the shortest path is by weight, not the number of edges.) Suggest a simple change to the Bellman-Ford algorithm that allows it to terminate in $m+1$ passes.

4. (1 Point) p.600: 24.3-1
   Run Dijkstra's algorithm on the directed graph of Figure 24.2, first using vertex s as the source and then using vertex z as the source. In the style of Figure 24.6, show the d and π values and the vertices in set S after each iteration of the **while** loop.



5. (10 Points) p.600: 24.3-4
We are given a directed graph $G = (V, E)$ on which each edge $(u, v) \in E$ has an associated value $r(u, v)$, which is a real number in the range $0 \le r(u, v) \le 1$ that represents the reliability of a communication channel from vertex u to vertex v. We interpret $r(u, v)$ as the probability that the channel from u to v will not fail, and we assume that these probabilities are independent. Give an efficient algorithm to find the most reliable path between two given vertices.

6. (1 Point) p.613: 24.5-4
Let $G = (V, E)$ be a weighted, directed graph with source vertex s and let G be initialized by INITIALIZE-SINGLE-SOURCE(G, s). Prove that if a sequence of relaxation steps sets $\pi[s]$ to a non-NIL value, then G contains a negative-weight cycle.