CMPS 101
Fall 2007
Homework Assignment 8

1. (1 Point) p.1091: B.5-4
   Use induction to show that a nonempty binary tree with n nodes has height at least $\lfloor \lg n \rfloor$.

2. (1 Point) p.132: 6.2-5
   The code for Max-Heapify is quite efficient in terms of constant factors, except possibly for the recursive call in line 10, which might cause some compilers to produce inefficient code. Write an efficient Max-Heapify that uses an iterative control construct (a loop) instead of recursion.

3. (1 Point) p.136: 6.4-2
   Argue the correctness of Heapsort using the following invariant:

   At the start of each iteration of the for loop on lines 2-5, the subarray $A[1\cdots i]$ is a max-heap containing the i smallest elements of $A[1\cdots n]$, and the subarray $A[(i+1)\cdots n]$ contains the $n-i$ largest elements of $A[1\cdots n]$ in increasing order.

   We reproduce the Heapsort pseudo-code below for reference:

   HeapSort(*A*)
   1. BuildMaxHeap(*A*)
   2. for $i \leftarrow$ length[*A*] down to 2
   3. $A[1] \leftrightarrow A[i]$
   4.   heap-size[*A*] $\leftarrow$ (heap-size[*A*] $-$ 1)
   5.   MaxHeapify(*A*, 1)