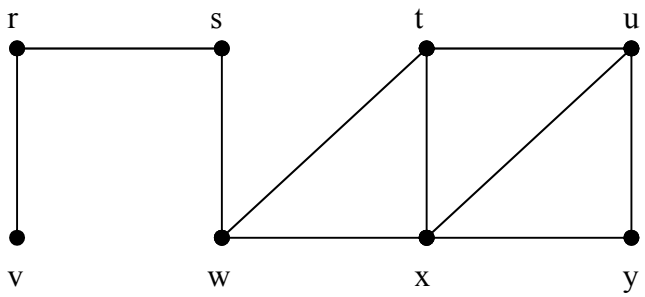
CMPS 101
Spring 2008
Homework Assignment 6

1. (10 Points)
   Let G be a forest (i.e. an acyclic graph) with n vertices, m edges, and k connected components. Show that $m = n - k$. (Hint: use the following fact which was proved in the induction handout: if T is a tree, then $|E(T)| = |V(T)| - 1$.)

2. (1 Point) p.75: 4.3-2
   The recurrence $T(n) = 7T(n/2) + n^2$ describes the running time of an algorithm A. A competing algorithm A' has a running time of $T'(n) = aT'(n/4) + n^2$. What is the largest integer value for a such that A' is asymptotically faster than A ?

3. (1 Point) p. 538: 22.2-2
   Show the d and π values that result from running breadth-first search on the undirected graph of Figure 22.3, using vertex u as the source.



4. (1 Point) p. 538: 22.2-6
   There are two types of professional wrestlers: "good guys" and "bad guys." Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have n professional wrestlers and we have a list of r pairs of wrestlers for which there are rivalries. Give an $O(n+r)$-time algorithm that determines whether it is possible to designate some of the wrestlers as good guys and the remainder as bad guys such that each rivalry is between a good guy and a bad guy. If it is possible to perform such a designation, your algorithm should produce it.

5. (1 Point) p.547: 22.3-1
   Make a 3-by-3 chart with row and column labels WHITE, GRAY, and BLACK. In each cell (i, j), indicate whether, at any point during a depth-first search of a directed graph, there can be an edge from a vertex of color i to a vertex of color j. For each possible edge, indicate what types it can be.