

CMPS 101
Spring 2008
Homework Assignment 4

1. (10 Points)

Let $f(n)$ be a positive, increasing function that satisfies $f(n/2) = \Theta(f(n))$. Show that

$$\sum_{i=1}^n f(i) = \Theta(nf(n))$$

(Hint: follow the example on page 4 of the handout on asymptotic growth rates.)

2. (10 Points)

Let $T(n)$ be defined by the recurrence formula

$$T(n) = \begin{cases} 1 & n = 1 \\ T(\lfloor n/2 \rfloor) + n^2 & n \geq 2 \end{cases}$$

Show that $\forall n \geq 1: T(n) \leq \frac{4}{3}n^2$, and hence $T(n) = O(n^2)$. (Hint: follow example 3 on page 3 of the handout on induction proofs.)

3. (1 Point) (Appendix B.4 problem 3)

Show that any connected graph $G = (V, E)$ satisfies $|E| \geq |V| - 1$, where $|E|$ and $|V|$ denote the number of edges and vertices, respectively.