CMPS 101
Spring 2008
hw3

1. (10 Points)

   Let $f(n) = \Theta(n)$. Prove that $\sum_{i=1}^{n} f(i) = \Theta(n^2)$. (See the hint at bottom of p.4 of the handout on asymptotic growth rates.)

2. (10 Points)
   Consider the statement $f(n/2) = \Theta(f(n))$. Determine whether this statement is (i) true for all functions $f(n)$, (ii) false for all functions $f(n)$, or (iii) true for some functions and false for some functions.

3. (1 Point) The last exercise in the handout entitled *Some Common Functions*.

   Use Stirling's formula to prove that $\binom{2n}{n} = \Theta\left(\frac{4^n}{\sqrt{n}}\right)$.

4. (2 Points) (Exercise 1 from the induction handout)

   Prove that for all $n \geq 1$: $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$. Do this twice:

   a. (1 Point) using form **II**a of the induction step.
   b. (1 Point) using form IIb of the induction step.

5. (1 Point) Exercise 2 from the induction handout)
   Define $S(n)$ for $n \in Z^+$ by the recurrence:

$$S(n) = \begin{cases} 0 & \text{if } n = 1 \\ S(\lceil n/2 \rceil) + 1 & \text{if } n \geq 2 \end{cases}$$

   Prove that $S(n) \geq \lg(n)$ for all $n \geq 1$, and hence $S(n) = \Omega(\lg n)$.