CMPS 101
Final Review Problems

1. Let T be a binary tree, and let $n(T)$ and $h(T)$ denote its number of nodes and height, respectively. Show that $h(T) \geq \lfloor \lg(n(T)) \rfloor$. (Hint: this was proved in the solutions to hw7.)

2. State the following properties:
   a. The max-Heap property
   b. The min-Heap property

3. Re-write the algorithms Heapify, and HeapIncreaseKey from the point of view of a min-Heap, rather than a max-Heap. (In particular, HeapIncreaseKey should be renamed HeapDecreaseKey.)

4. Let T be an almost complete binary tree on n nodes, and let $N(n, h)$ denote the number of nodes in T at height h, where $0 \leq h \leq \lfloor \lg(n) \rfloor$. Prove that $N(n, h) = \left\lfloor \frac{n}{2^h} \right\rfloor - \left\lfloor \frac{n}{2^{h+1}} \right\rfloor$. Hint: first observe that the number of leaves in T is $n - \left\lfloor \frac{n}{2} \right\rfloor$. (Note this is a problem from hw7.)

5. Trace HeapSort on the following arrays:
   a. (9, 3, 5, 4, 8, 2, 5, 10, 12, 2, 7, 4)
   b. (5, 3, 7, 1, 10, 12, 19, 24, 5, 7, 2, 6)
   c. (9, 8, 7, 6, 5, 4, 3, 2, 1)

6. Let $x \in V(G)$, and suppose that Initialize(G, s) is called. Show that the inequality $\delta(s, x) \leq d[x]$ is maintained over *any* sequence of calls to Relax(,). (Use the following result from problem 4 in homework assignment 7: If after Initialize(G, s) is executed some Relaxation sequence causes $d[x]$ to become finite, then G contains an s-x path of weight $d[x]$.)