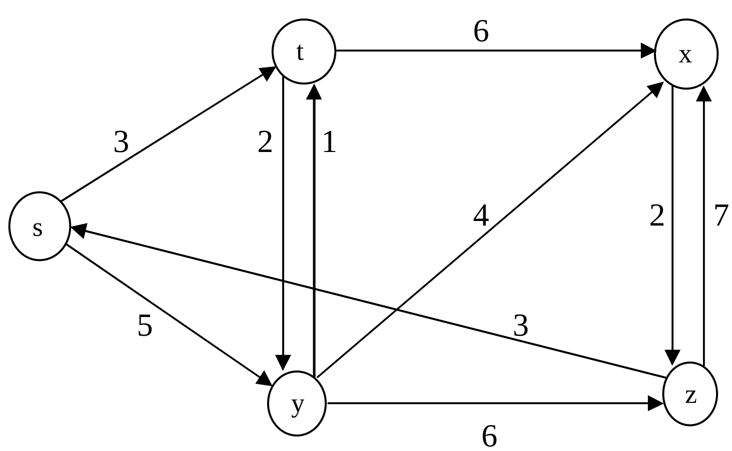
CMPS 101
Fall 2009
Homework Assignment 8

1. (1 Point) p.132: 6.2-5
   The code for Max-Heapify is quite efficient in terms of constant factors, except possibly for the recursive call in line 10, which might cause some compilers to produce inefficient code. Write an efficient Max-Heapify that uses an iterative control construct (a loop) instead of recursion.

2. (1 Point)
   Let $x \in V(G)$ and suppose that after INITIALIZE-SINGLE-SOURCE(G, s) is executed, some sequence of calls to Relax( ) causes $d[x]$ to be set to a finite value. Then G contains an s-x path of weight $d[x]$. (Hint: Use induction on the length of the Relaxation sequence, and recall that this result was proved in class.)

3. (1 Point) p.591: 24.1-3
   Given a weighted, directed graph $G = (V, E)$ with no negative-weight cycles, let m be the maximum over all pairs of vertices $u, v \in V$ of the minimum number of edges in a shortest path from u to v. (Here, the shortest path is by weight, not the number of edges.) Suggest a simple change to the Bellman-Ford algorithm that allows it to terminate in $m+1$ passes.

4. (1 Point) p.600: 24.3-1
   Run Dijkstra's algorithm on the directed graph of Figure 24.2, first using vertex s as the source and then using vertex z as the source. In the style of Figure 24.6, show the d and π values and the vertices in set S after each iteration of the **while** loop.



5. (1 Points) p.600: 24.3-4
   We are given a directed graph $G = (V, E)$ on which each edge $(u, v) \in E$ has an associated value $r(u, v)$, which is a real number in the range $0 \le r(u, v) \le 1$ that represents the reliability of a communication channel from vertex u to vertex v. We interpret $r(u, v)$ as the probability that the channel from u to v will not fail, and we assume that these probabilities are independent. Give an efficient algorithm to find the most reliable path between two given vertices.

6. (1 Point) p.613: 24.5-4
Let $G = (V, E)$ be a weighted, directed graph with source vertex s and let G be initialized by INITIALIZE-SINGLE-SOURCE(G, s). Prove that if a sequence of relaxation steps sets $\pi[s]$ to a non-NIL value, then G contains a negative-weight cycle.