

CMPS 101

Fall 2009

Homework Assignment 7

1. (1 Point) p.551: 22.4-1
Show the ordering of vertices produced by TOPOLOGICAL-SORT when it is run on the dag of Figure 22.8, under the assumption of Exercise 22.3-2.
2. (1 Point) p.557: 22.5-2
Show how the procedure STRONGLY-CONNECTED-COMPONENTS works on the graph of Figure 22.6. Specifically, show the finishing times computed in line 1 and the forest produced in line 3. Assume that the loop of lines 5-7 of DFS considers vertices in alphabetical order and that the adjacency lists are in alphabetical order.
3. (1 Point) p.1091: B.5-4
Use induction to show that a nonempty binary tree with n nodes has height at least $\lfloor \lg n \rfloor$. Hint: use the recursive definition of height discussed in class:

$$h(T) = \begin{cases} -\infty & n(T) = 0 \\ 0 & n(T) = 1 \\ 1 + \max(h(L), h(R)) & n(T) > 1 \end{cases}$$

Here $n(T)$ denotes the number of nodes in a binary tree T , $h(T)$ denotes its height, L denotes its left subtree, and R its right subtree. Note that this proof can be phrased equally well as an induction on $n(T)$ or on $h(T)$. (Hint: Use (and prove) the following fact: $\lfloor \lg(2k+1) \rfloor = \lfloor \lg(2k) \rfloor$ for any positive integer k .)