CMPS 101
Fall 2009
Homework Assignment 4

1. (3 Points)
   Consider the function $T(n)$ defined by the recurrence formula
   $$T(n) = \begin{cases} 6 & 1 \le n < 3 \\ 2T(\lfloor n/3 \rfloor) + n & n \ge 3 \end{cases}$$
   a. (1 Point) Use the iteration method to write a summation formula for $T(n)$.
   b. (1 Point) Use the summation in (a) to show that $T(n) = O(n)$
   c. (1 Point) Use the Master Theorem to show that $T(n) = \Theta(n)$

2. (6 Points)
   Use the Master theorem to find asymptotic solutions to the following recurrences.
   a. (1 Point) $T(n) = 7T(n/4) + n$
   b. (1 Point) $T(n) = 9T(n/3) + n^2$
   c. (1 Point) $T(n) = 6T(n/5) + n^2$
   d. (1 Point) $T(n) = 6T(n/5) + n\log(n)$
   e. (1 Point) $T(n) = 7T(n/2) + n^2$
   f. (1 Point) $S(n) = aS(n/4) + n^2$ (Note: your answer will depend on the parameter a.)

3. (1 Point) p.75: 4.3-2
   The recurrence $T(n) = 7T(n/2) + n^2$ describes the running time of an algorithm *A*. A competing algorithm *B* has a running time of $S(n) = aS(n/4) + n^2$. What is the largest integer value for a such that *B* is a faster algorithm than *A* (asymptotically speaking)? In other words, find the largest integer a such that $S(n) = o(T(n))$.

4. (1 Points)
   Let G be an acyclic graph with n vertices, m edges, and k connected components. Show that $m = n - k$. (Hint: use the fact that $|E(T)| = |V(T)| - 1$ for any tree *T*, from the induction handout.)

5. (1 Point) (Appendix B.4 problem 3)
   Show that any connected graph G satisfies $|E(G)| \ge |V(G)| - 1$. (Hint: use induction on the number of edges.)