

CMPS 101
Fall 2009
Homework Assignment 2

1. (1 Point) p.50: 3.1-1

Let $f(n)$ and $g(n)$ be asymptotically non-negative functions. Using the basic definition of Θ -notation, prove that $f(n) + g(n) = \Theta(\max(f(n), g(n)))$.

2. (1 Point) p.50: 3.1-3

Explain why the statement “The running time of algorithm A is at least $O(n^2)$ ” is meaningless.

3. (2 Points) p. 50: 3.1-4

Determine whether the following statements are true or false.

a. (1 Point) $2^{n+1} = O(2^n)$

b. (1 Point) $2^{2n} = O(2^n)$

4. (6 Points) p.58: 3-2abcdef

Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, Ω, ω , or Θ of B . Assume that $k \geq 1$, $\varepsilon > 0$, and $c > 1$ are constants. Place 'yes' or 'no' in each of the empty cells below, and justify your answers.

	A	B	O	o	Ω	ω	Θ
a. (1 Point)	$\lg^k n$	n^ε					
b. (1 Point)	n^k	c^n					
c. (1 Point)	\sqrt{n}	$n^{\sin n}$					
d. (1 Point)	2^n	$2^{n/2}$					
e. (1 Point)	$n^{\lg c}$	$c^{\lg n}$					
f. (1 Point)	$\lg(n!)$	$\lg(n^n)$					

5. (4 Points) p.58: 3-4cdeh

Let $f(n)$ and $g(n)$ be asymptotically positive functions (i.e. $f(n) > 0$ and $g(n) > 0$ for sufficiently large n .) Prove or disprove the following statements.

c. (1 Point)

Assume $\lg(g(n)) \geq 1$ and $f(n) \geq 1$ for all sufficiently large n . Then $f(n) = O(g(n))$ implies $\lg(f(n)) = O(\lg(g(n)))$.

d. (1 Point)

$f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$.

e. (1 Point)

$f(n) = O((f(n))^2)$.

h. (1 Point)

$f(n) + o(f(n)) = \Theta(f(n))$.

6. (1 Point)

Let $f(n) = \Theta(n)$. Prove that $\sum_{i=1}^n f(i) = \Theta(n^2)$. (See the hint at bottom of p.4 of the handout on asymptotic growth rates.)