CMPS 101
Midterm 2 Review Problems

Figure 1:

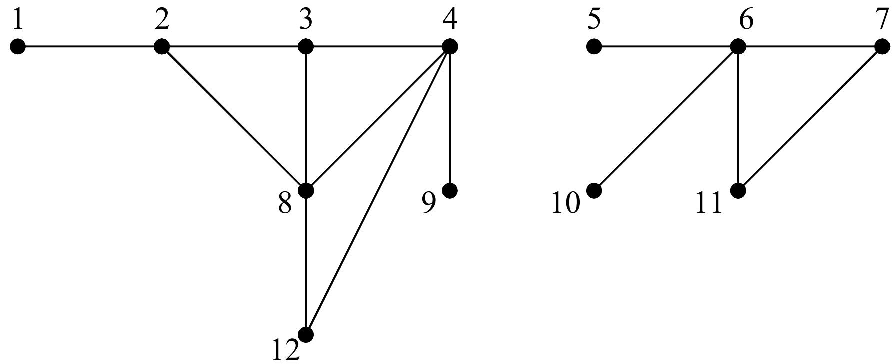


Figure 2:

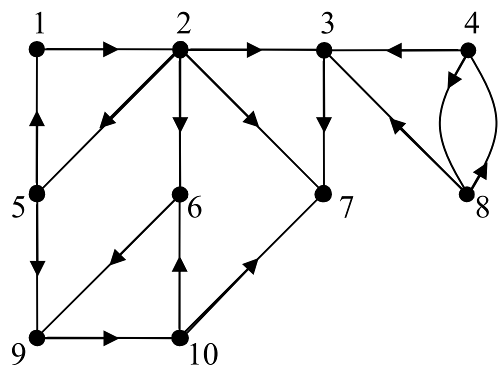
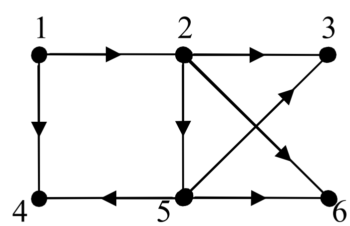


Figure 3:



Problems

1. Trace BFS on the following graphs. For each vertex, record its color, parent, and distance fields, and draw the resulting BFS tree. Process adjacency lists in (ascending) numerical order.
   a. The graph in figure 1, with 1 as the source.
   b. The directed graph in figure 2 with 1 as source.

2. Trace DFS on the following graphs. For each vertex, record its color, parent, discover, and finish fields, and draw the resulting DFS forest. Classify each edge as tree, back forward, or cross. Process adjacency lists in (ascending) numerical order.
   a. The graph in figure 1. Process vertices in the main loop of DFS in (ascending) numerical order.
   b. The graph in figure 2. Process vertices in the main loop of DFS in (ascending) numerical order.
   c. The transpose of the graph in figure 2. Process vertices in the main loop of DFS in order of descending finish times from part b. Determine the strongly connected components of the graph in figure 2, and draw its component graph.
   d. The graph in figure 3. Process vertices in the main loop of DFS in (ascending) numerical order. Show that this graph is acyclic and determine a topological sort of the vertices.
   e. The graph in figure 3. Process vertices in the main loop of DFS in descending order. Determine a topological sort of the vertices which is different from that in part d.

3. Write and algorithm called isBipartite(G) which takes as input a connected (undirected) graph G and returns *true* or *false* according to whether or not G is bipartite. (Hint: see the solutions to hw problem 22.2-6 for the definition of a bipartite graph.)

4. Let $G=(V,E)$ be a connected (undirected) graph. Prove $|E|\geq|V|-1$. (Hint: use induction on $|E|$.)

5. Let G be a directed graph. Determine whether, at any point during a Depth First Search of G, there can exist an edge of the following kind.
   a. A tree edge which joins a white vertex to a gray vertex.
   b. A back edge which joins a black vertex to a white vertex.
   c. A forward edge which joins a gray vertex to a black vertex.
   d. A cross edge which joins a black vertex to a gray vertex.

6. a. State the parenthesis theorem.
   b. State the white path theorem.

7. Let G be a directed graph. Prove that if G contains a directed cycle, then G contains a back edge. (Hint: use the white path theorem.)

8. Let $G=(V,E)$ be a weighted graph, and let $x\in V$. Suppose that after Initialize(G, s) has been called, some sequence of calls to Relax(,) causes $d[x]$ to be set to a finite value. Show that G contains an s-x path of weight $d[x]$. (Hint: use induction on the length n of the relaxation sequence.)

9. Let $G=(V,E)$ be a weighted graph. Show that the inequality $\delta(x,s)\leq d[x]$ is maintained over any sequence of calls to Relax(,). (Hint: use the result of problem 8 and proof by contradiction.)