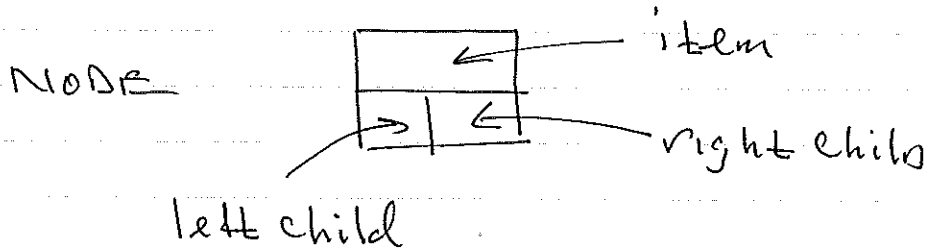


CHD 11 - BINARY SEARCH TREES -

A BINARY SEARCH TREE IS A LINKED DATA STRUCTURE, IN SOME WAYS ANALOGOUS TO A LINKED LIST.



WE CAN CREATE SUCH A NODE TYPE IN C BY

```

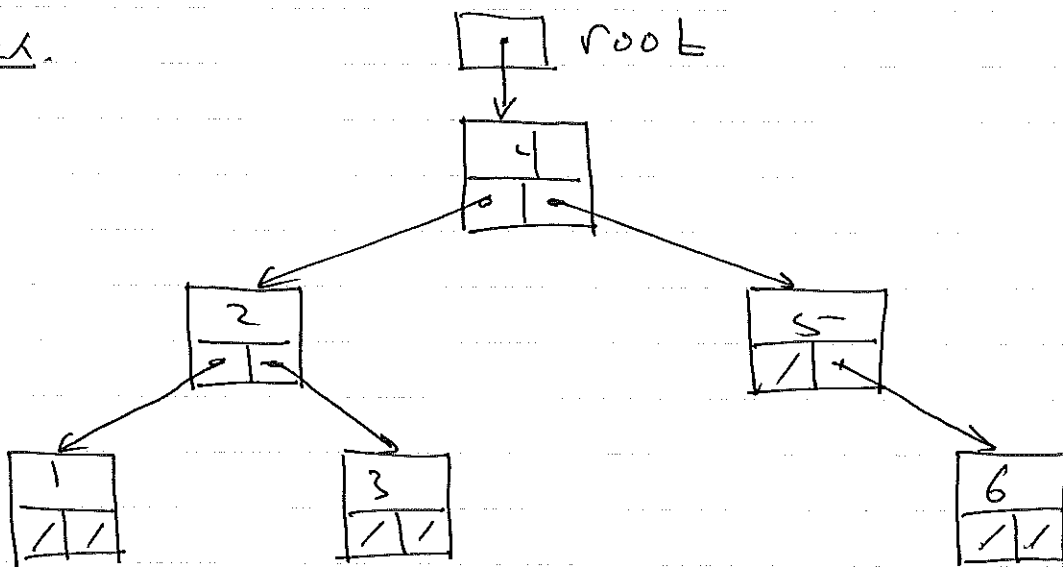
typedef struct Node {
    int item;
    struct Node* left;
    struct Node* right;
} Node;
  
```

AND A REFERENCE TO THIS NODE BY

```

typedef Node* NodeRef;
  
```

EX.



Binary Searched Tree Properties:

LET x AND y BE OF TYPE NodeRef:

- IF y IS IN THE LEFT SUBTREE OF x
THEN $y \rightarrow \text{item} \leq x \rightarrow \text{item}$
- IF y IS IN THE RIGHT SUBTREE OF x
THEN $x \rightarrow \text{item} \leq y \rightarrow \text{item}$

THE BST PROPERTIES MAKES IT POSSIBLE TO TRAVEL THE TREE IN SEVERAL ORDERS:

```
void printInOrder(NodeRef x) {
```

```
    if (x != NULL) {
```

```
        printInOrder(x->left);
```

```
        printf("%d", x->item);
```

```
        printInOrder(x->right);
```

```
    }
```

```
}
```

CALL: `printInOrder(root)`

OUTPUT FROM LAST EX:

1 2 3 4 5 6

```

void PrintPreOrder(NodeKet x) {
  if (x != NULL) {
    printf("%d ", x->item);
    PrintPreOrder(x->left);
    PrintPreOrder(x->right);
  }
}

```

CALL: PrintPreOrder(root);

OUTPUT: 4 2 1 3 5 6

```

void PrintPostOrder(NodeKet x) {
  if (x != NULL) {
    PrintPostOrder(x->left);
    PrintPostOrder(x->right);
    printf("%d ", x->item);
  }
}

```

CALL: PrintPostOrder(root)

OUTPUT: 1 3 2 6 5 4

THE BST PROPERTIES ALLOW ONE TO SEARCH FOR A GIVEN TARGET EFFICIENTLY. THE FOLLOWING FUNCTION RETURNS A REFERENCE TO THE NODE IN THE SUBTREE ROOTED AT R WITH ITEM K , OR NULL IF NO SUCH NODE EXISTS.

```
NodeRef findItem(NodeRef R, int k) {
    if (R == NULL || k == R->item)
        return R;
    if (k < R->item)
        return findItem(R->left, k);
    else
        return findItem(R->right, k);
}
```

TO SEARCH THE ENTIRE TREE WE CALL

$\text{findItem}(\text{root}, k)$;

WHERE root IS A NodeRef POINTING TO THE ROOT NODE.

THE RUNNING TIME OF findItem IS $O(h)$ WHERE h IS THE HEIGHT OF THE BST, IN WORST CASE.

OTHER OPERATIONS WHICH CAN BE PERFORMED EFFICIENTLY ARE findMin AND findMax WHICH RETURN THE NODES WITH MINIMUM AND MAXIMUM KEYS, RESPECTIVELY

```

NodeRef findMin(NodeRef R) {
  NodeRef L = R;
  if (L != NULL) for( ; L->left != NULL; L = L->left);
  return L;
}

```

This simply returns the leftmost node in the subtree rooted at R , or NULL if R is NULL.

A BST can be used to implement the Dictionary ADT from PA2 and LAB5. The essential Dictionary operations are

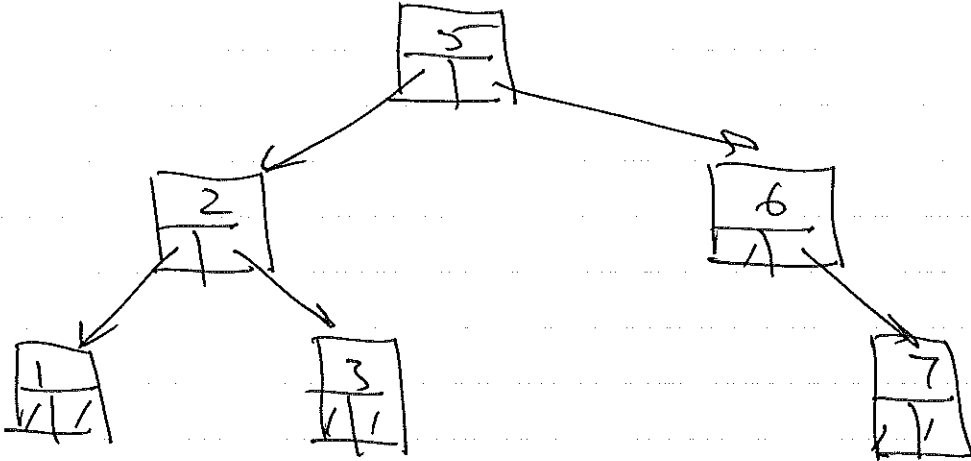
- Insert
- Delete
- Lookup

Lookup can be based on findItem above.

To do insertion we do something very similar to lookup. Let NodeRef A initially point to the root, and NodeRef B be NULL. Allow A to climb down the tree to the NULL child position where the new key belongs, with B maintained as A 's parent. Insert the new node as B 's child.

Ex

Insert (4):



A Deletion is considerably more complicated. We have 3 cases.

- 1: N has no children
- 2: N has 1 child
 - 2.1 left
 - 2.2 right
- 3: N has 2 children.

See Dictionary.

Dictionary.h

```

/*
 * Dictionary.h
 * Header file for the Dictionary ADT
 */

#ifndef _DICTIONARY_H_INCLUDE_
#define _DICTIONARY_H_INCLUDE_

#define UNDEF -1

/*
 * DictionaryRef
 * Exported reference type which points to a Dictionary struct
 */
typedef struct Dictionary* DictionaryRef;

/*
 * newDictionary
 * Allocates and initializes a struct representing an empty Dictionary. Returns a
 * pointer to new memory if successful, terminates if not successful.
 */
DictionaryRef newDictionary(void);

/*
 * freeDictionary
 * Frees all heap memory associated with *pD and sets the reference *pD to NULL
 */
void freeDictionary(DictionaryRef* pD);

/*
 * isEmpty
 * pre: none
 * post: returns 1 (true) if D is empty, 0 (false) otherwise
 */
int isEmpty(DictionaryRef D);

/*
 * size
 * pre: none
 * post: returns the number of entries in D
 */
int size(DictionaryRef D);

/*
 * lookup
 * pre: none
 * post: returns value associated key k, or UNDEF if no such key exists
 */
int lookup(DictionaryRef D, int k);

/*
 * insert
 * inserts new (key,value) pair into D
 * pre: key k currently does not exist in D, i.e. lookup(D, k)==UNDEF
 * post: !isEmpty(D), size(D) is increased by one
 */
void insert(DictionaryRef D, int k, int v);

/*
 * delete
 * deletes pair with the key k
 * pre: key k currently exists in D, i.e. lookup(D, k)!=UNDEF
 * post: size(D) is decreased by one
 */
void delete(DictionaryRef D, int k);

```