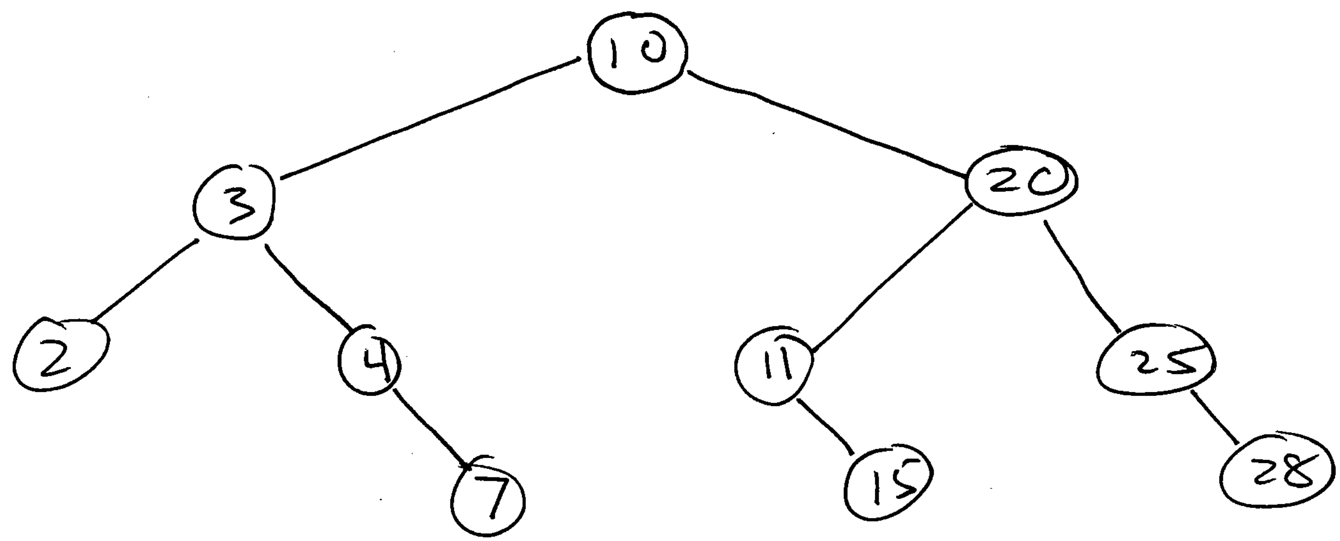
CMPS 10 2-7-08

Algorithm	Runtime (worst case)	Asymptotic Runtime
SEQ. SEARCH	n	$\Theta(n)$
SELECT. SORT	$\frac{n(n-1)}{2} = \frac{1}{2}n^2 - \frac{1}{2}n$	$\Theta(n^2)$
Bubble. SORT	$\frac{n(n-1)}{2} = \frac{1}{2}n^2 - \frac{1}{2}n$	$\Theta(n^2)$
Insert. Sort	$\frac{n(n-1)}{2} = \frac{1}{2}n^2 - \frac{1}{2}n$	$\Theta(n^2)$
Binary Search	?	?

EX.

Pos:	1	2	3	4	5	6	7	8	9	10
	2	3	4	7	10	11	15	20	25	28



 # COMP

1
2
3
4

WORST CASE # COMP $= 4$

BEST CASE # COMP $= 1$

AVG CASE # COMP $= \dfrac{1 + 2\cdot 2 + 4\cdot 3 + 3\cdot 4}{10} = \dfrac{29}{10} = 2.9$

(ASSUME target in list, All POS. EQUALLY likely.

<u>NOTATION</u>: LET $W(n)$ DENOTE THE WORST CASE # OF COMPARISONS PERFORMED BIN. SEARCH ON LISTS OF LENGTH n.

ASSUME FROM NOW ON THAT THE LIST THAT WE'RE SEARCHING IS:

$$1 \quad 2 \quad 3 \quad \cdots\cdots \quad n$$

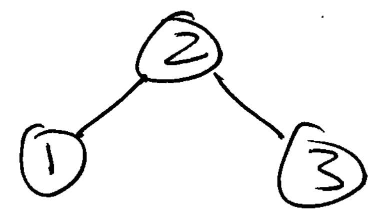
<u>GOAL</u>: FIND A FORMULA FOR $W(n)$

4

<u>$n=3$</u>:

$3 = 2^2 - 1$

1 2 3

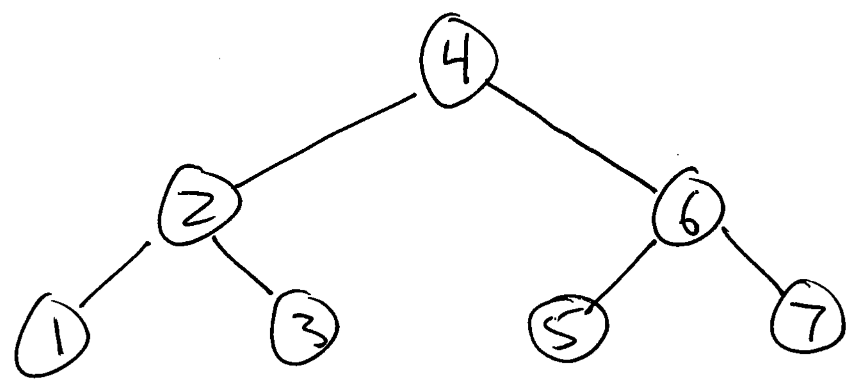


$w(3) = 2$

<u>$n=7$</u>:

$7 = 2^3 - 1$

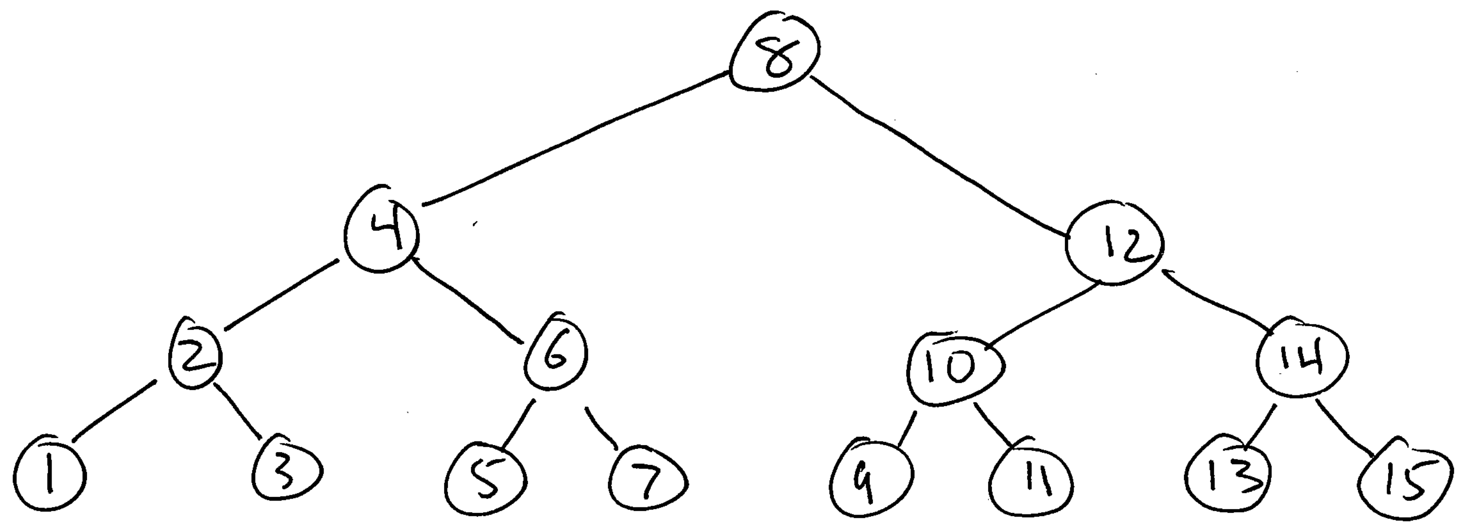
1 2 3 4 5 6 7



$w(7) = 3$

$\underline{n=15} = 2^4 - 1$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15



$w(15) = 4$

OBSERVE: IF $n = 2^k - 1$ THEN THE CORRESPONDING B.S.T. IS COMPLETE (i.e. EACH NODE HAS 0 OR 2 CHILDREN) AND HAS DEPTH k, WHENCE $w(n) = k$

<u>EXERCISE</u>: DRAW BSTs FOR lists OF length $n = 8, 9, 10, 11, 12, 13, 14$. OBSERVE THAT THEY ALL HAVE DEPTH 4. SO

$$w(8) = w(9) = w(10) = w(11) = w(12) = w(13) = w(14) = w(15) = 4$$

7

THEOREM

IF $2^{K-1} - 1 < n \leq 2^{K} - 1$ THEN THE #

OF COMP PERFORMED BY B.S. ON A LIST

OF length IS AT WORST: $W(n) = K$.

REALLY WANT A FORMULA FOR K.

DEFN:

LET $b > 1$, $x > 0$. WE DEFINE $\log_b(x)$ TO BE THE POWER YOU MUST RAISE b TO TO GET x

i.e. $\boxed{y = \log_b(x) \quad \text{IFF} \quad x = b^y}$

b: base OF THE log function.

EX.

$\log_3(9) = 2$ SINCE $3^2 = 9$

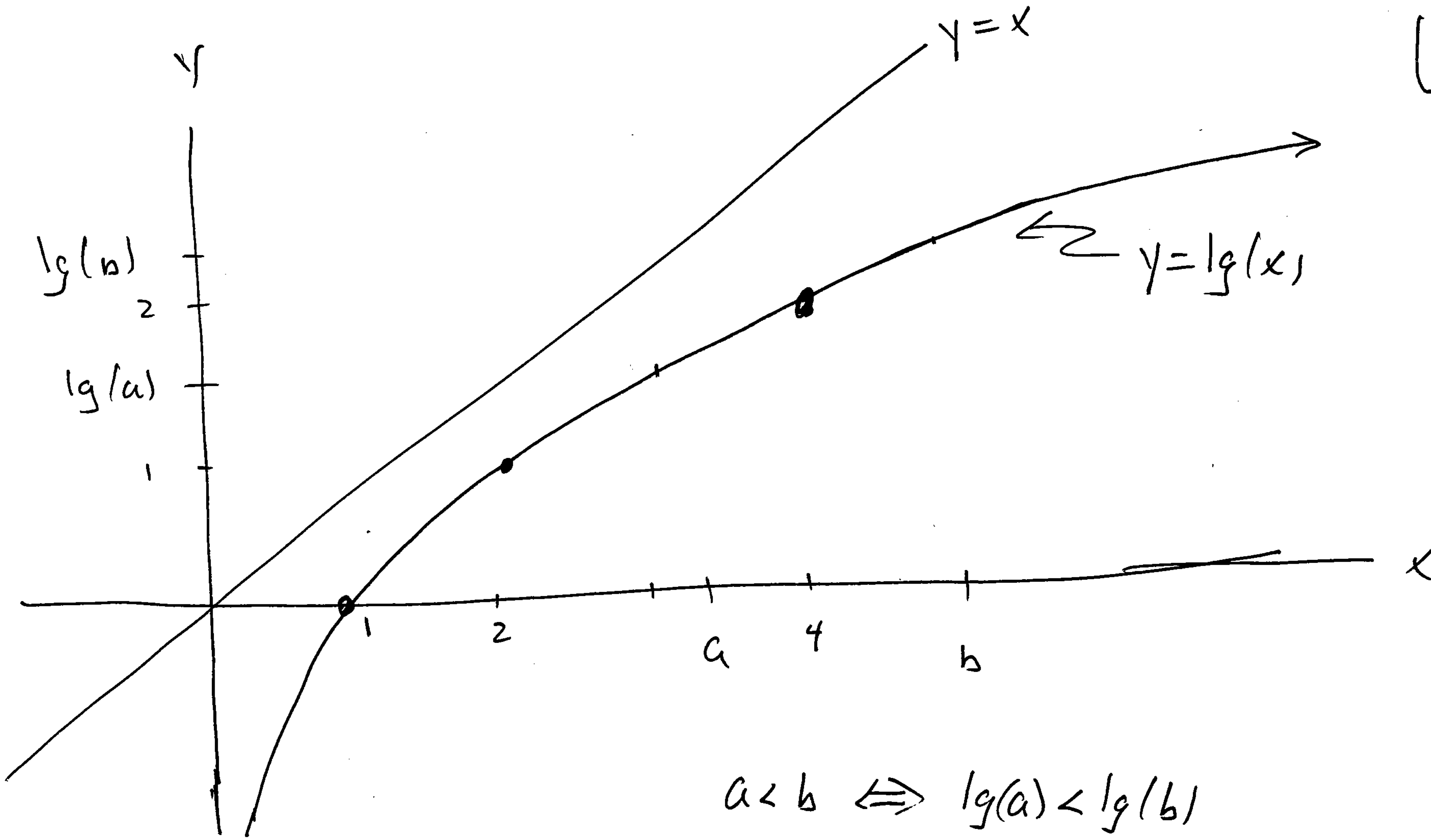
$\log_5(125) = 3$ " $5^3 = 125$

$\log_{10}(10000) = 4$ " $10^4 = 10000$

$\log_2(32) = 5$ " $2^5 = 32$

SPECIAL log FUNCTIONS!

			NOTATION
COMMON log or NAPIERIAN log	is	$\log_{10}(\cdot)$	$\log(\ )$
NATURAL log	is	$\log_e(\ )$	$\ln(\ )$
		$e = 2.71828\ldots$	
Binary log	is	$\log_2(\ )$	$\lg(\ )$



$a < b \iff \lg(a) < \lg(b)$

$\lg(1) = 0$

$\lg(2) = 1$

$\lg(4) = 2$

$\lg(2^K) = K$

__OBSERVE__: n GROWS __MUCH__ FASTER THAN $\lg(n)$ (OR $\log_b(n)$ FOR ANY b.)

__RECALL PREVIOUS THM__:

$$2^{k-1} - 1 < n \le 2^k - 1 \quad \text{IFF} \quad w(n) = k$$

MANIPULATE THIS ALGEBRAICALLY

$$2^{k-1} - 1 < n \le 2^k - 1 \iff 2^{k-1} \le n < 2^k$$

$$\iff \lg(2^{k-1}) \le \lg(n) < \lg(2^k)$$

$$\iff k-1 \le \lg(n) < k$$

$$\Longleftrightarrow \quad k-1 = \lfloor \lg(n) \rfloor$$

$$\Longleftrightarrow \quad k = \lfloor \lg(n) \rfloor + 1$$

THM: $\boxed{W(n) = \lfloor \lg(n) \rfloor + 1}$

CHECK for $n = 10$:

$$2^3 = 8 < 10 < 16 = 2^4$$

$$3 < \lg(10) < 4$$

so $\lfloor \lg(10) \rfloor > 3$

so $W(10) = \lfloor \lg(10) \rfloor + 1 = 3 + 1 = 4$

NOTE ALSO THAT $\lfloor f(n) \rfloor = \Theta(f(n))$

SINCE $\dfrac{\lfloor f(n) \rfloor}{f(n)} \to 1$ AS $n \to \infty$

SO $W(n) = \lfloor \lg(n) \rfloor + 1 = \Theta(\lg(n))$

<u>QUESTION</u>: IF HAVE UNSORTED LIST WHAT'S THE COST TO FIRST SORT, THEN SEARCH WITH B.S.

COST: $\Theta(n^2 + \lg n) = \Theta(n^2)$