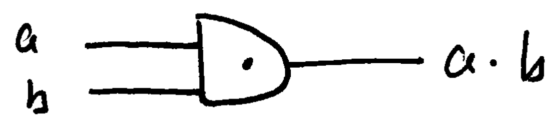
CMPS 10 2-19-08

Recall:

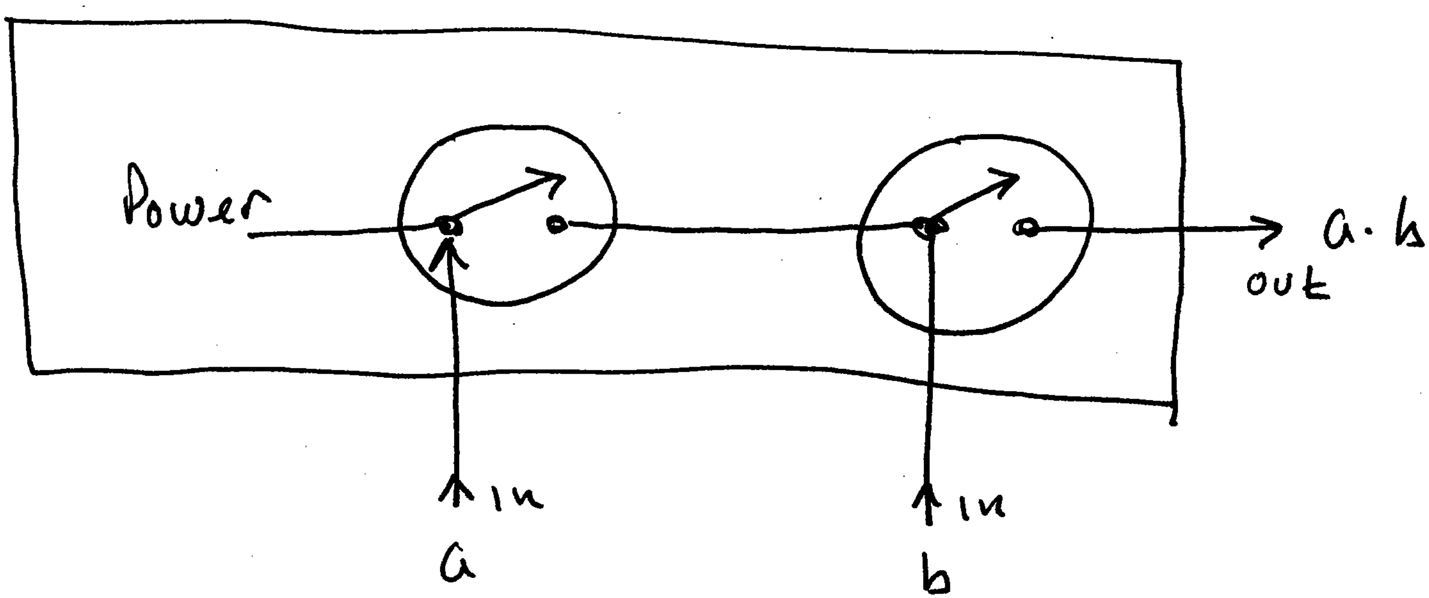
<u>LOGIC</u>		<u>MATH</u>		<u>CIRCUITS</u>
a and b	$\approx$	$a \wedge b$	$\approx$	$a \cdot b$
a or b	$\approx$	$a \vee b$	$\approx$	$a + b$
not a	$\approx$	$\neg a$	$\approx$	$\overline{a}$
a exor b	$\approx$	$a \overline{\vee} b$	$\approx$	$a \oplus b$

AND GATE

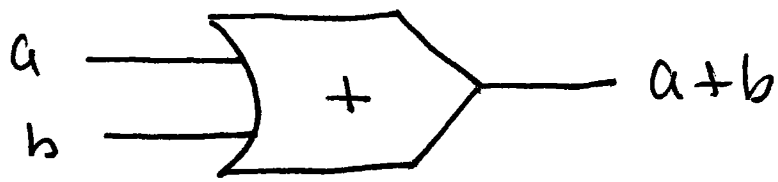


a	b	a · b
0	0	0
0	1	0
1	0	0
1	1	1

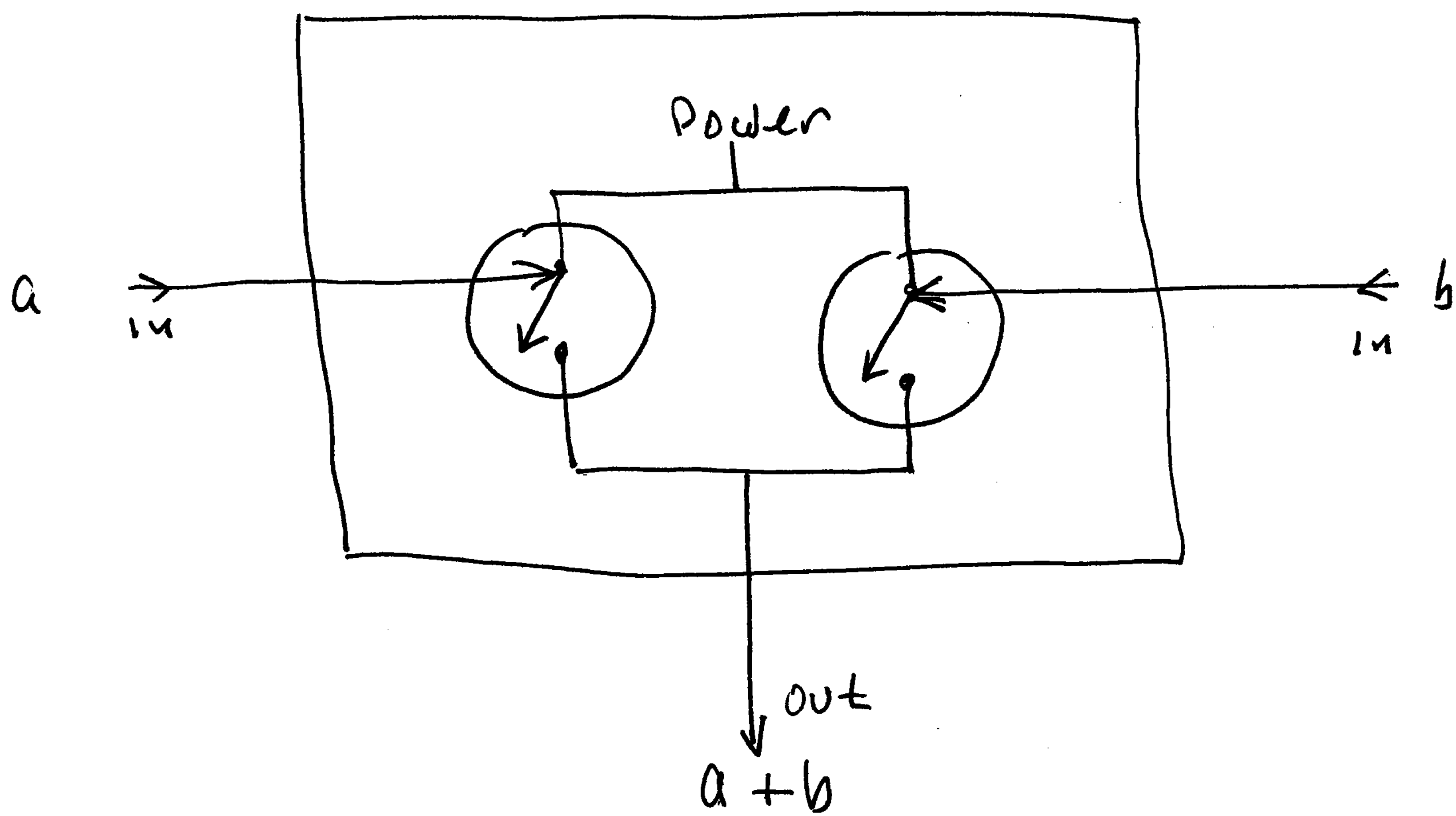
1 = true = high Voltage
0 = false = low Voltage



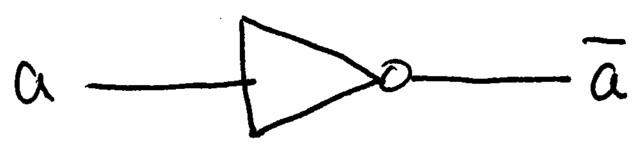
OR GATE



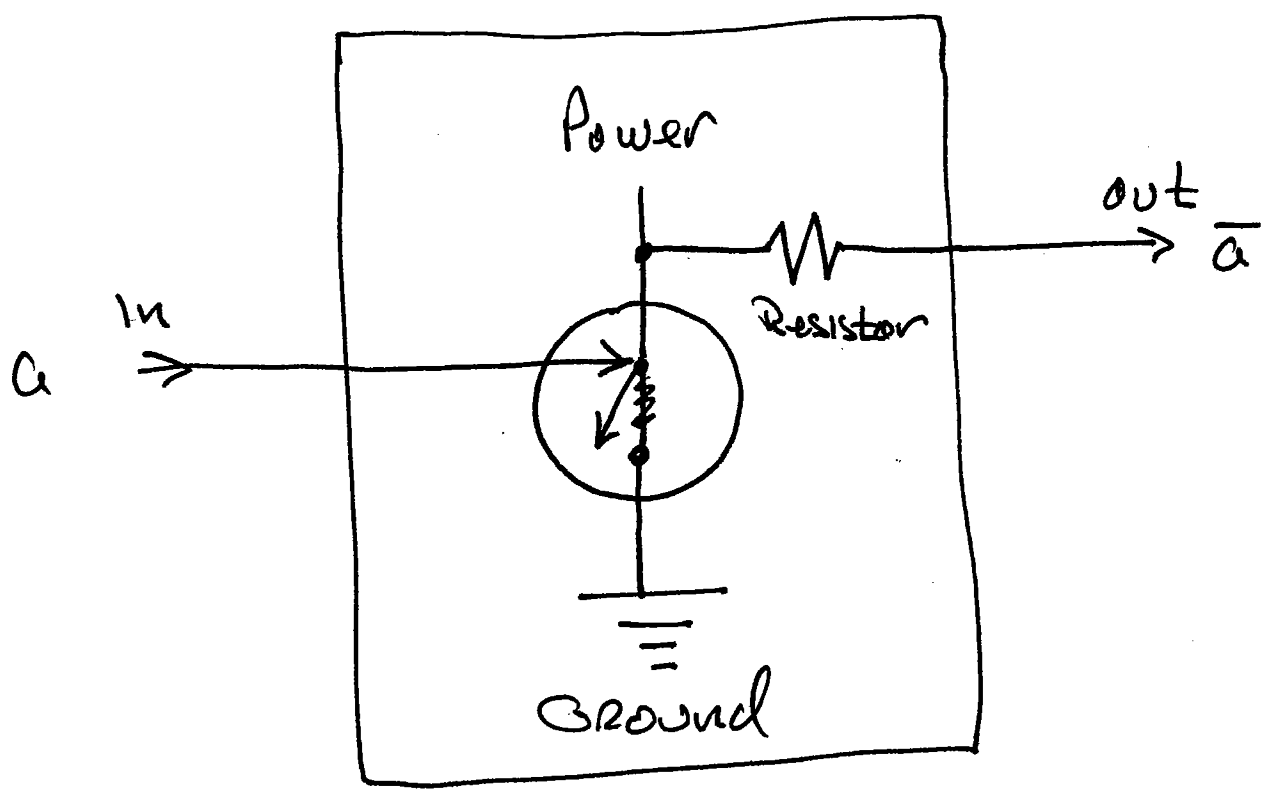
a	b	a + b
0	0	0
0	1	1
1	0	1
1	1	1



NOT GATE



a	$\overline{a}$
0	1
1	0

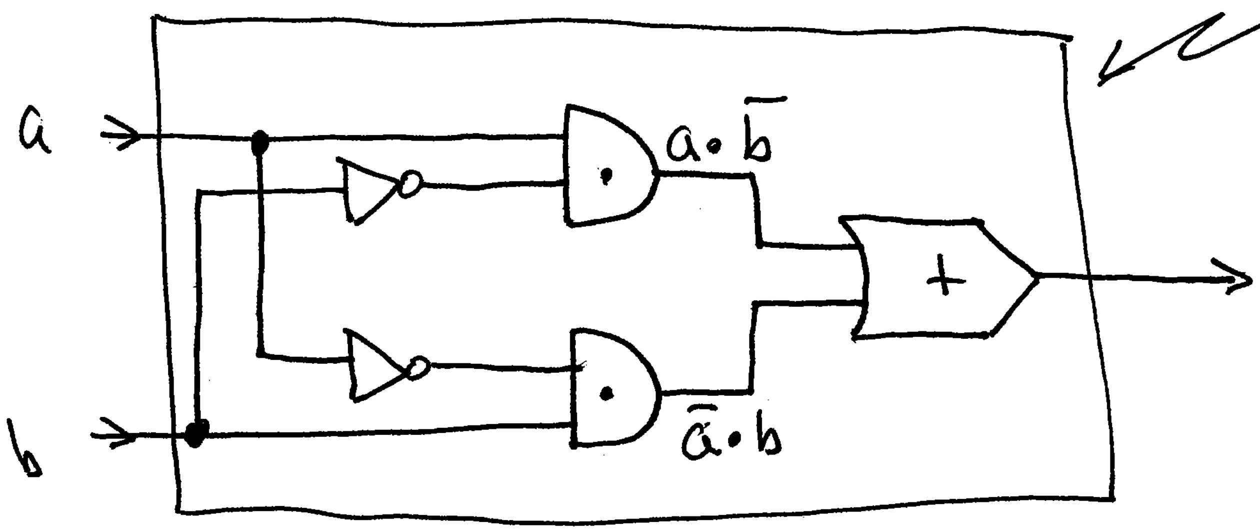


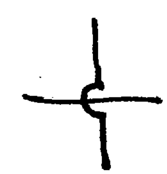
exor GATE: (from and, or, & not gates)

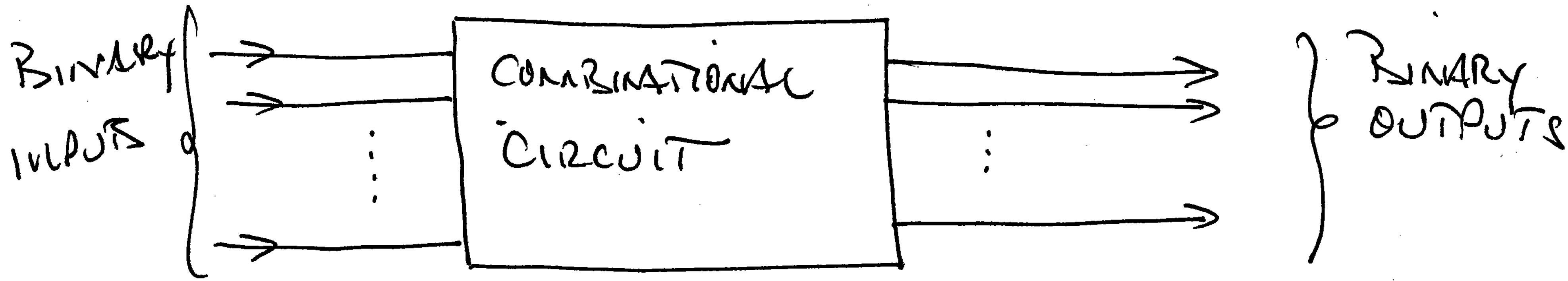
a	b	$a \oplus b$	$\bar{a}$	$\bar{b}$	$a \cdot \bar{b}$	$\bar{a} \cdot b$	$(a \cdot \bar{b}) + (\bar{a} \cdot b)$
0	0	0	1	1	0	0	0
0	1	1	1	0	0	1	1
1	0	1	0	1	1	0	1
1	1	0	0	0	0	0	0

$$a \oplus b \equiv (a \cdot \bar{b}) + (\bar{a} \cdot b)$$

exor gate

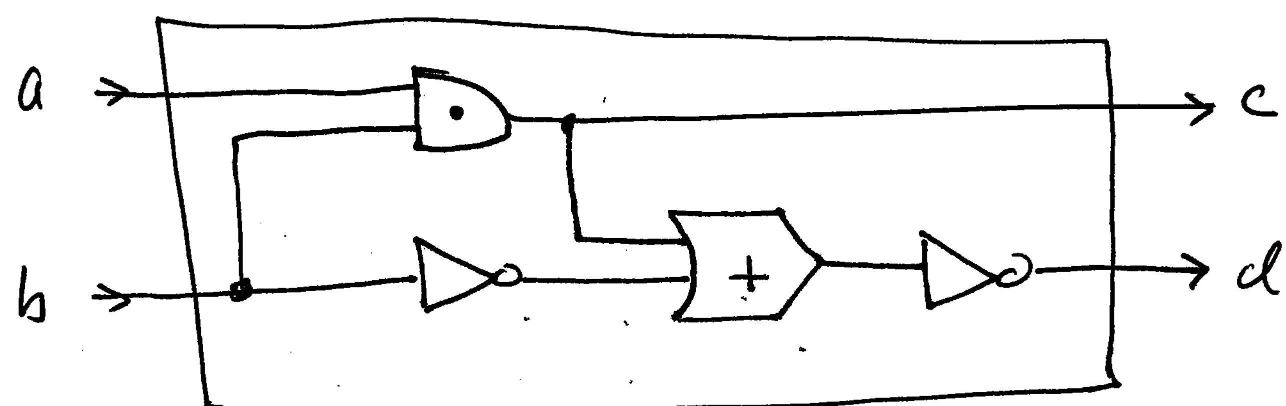






REQUIREMENT: OUTPUTS DEPEND ONLY ON CURRENT VALUES OF INPUTS

EX.



#Transistors = 6

TRUTH TABLE

a	b	c	d	$a \cdot b$	$\bar{b}$	$(a \cdot b) + \bar{b}$	$\overline{(a \cdot b) + \bar{b}}$
0	0	0	0	0	1	1	0
0	1	0	1	0	0	0	1
1	0	0	0	0	1	1	0
1	1	1	0	1	0	1	0

NOTE: $d \equiv \overline{(a \cdot b) + \bar{b}}$

LOGICAL EQUIVALENCE

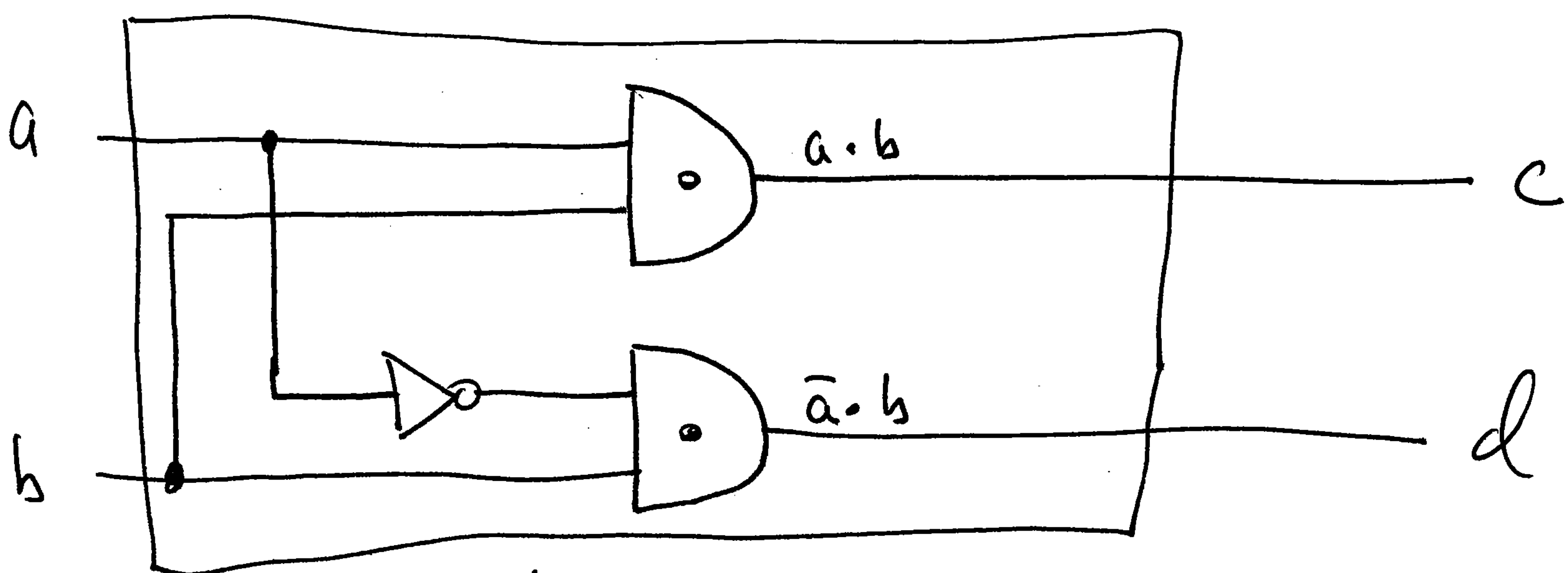
i.e. HAVE THE SAME TRUTH TABLE.

NOTE ALSO $d \equiv \bar{a} \cdot b$

a	b	$\bar{a}$	$\bar{a} \cdot b$
0	0	1	0
0	1	1	1
1	0	0	0
1	1	0	0

SO

$$\bar{a} \cdot b \equiv \overline{(a \cdot b) + \bar{b}}$$

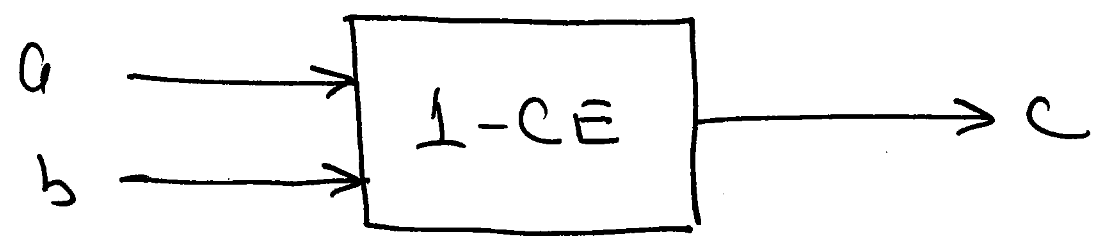


TRANSISTORS = 5

CORRESPONDENCES

$$\begin{tikzcd}[column sep=small]
\boxed{\text{CIRCUITS}} \arrow[rr, leftrightarrow, "\text{one-to-one}"] \arrow[dr, "\text{many-to-one}"'] & & \boxed{\text{LOGICAL EXPRESSIONS}} \arrow[dl, "\text{many-to-one}"] \\
& \boxed{\text{TRUTH TABLES}} &
\end{tikzcd}$$

<u>EX.</u> CONSTRUCT A 1-BIT COMP. FOR EQUALITY CIRCUIT — i.e. COMPARE BITS a & b. OUTPUT 1 IF $a = b$, 0 IF $a \neq b$.

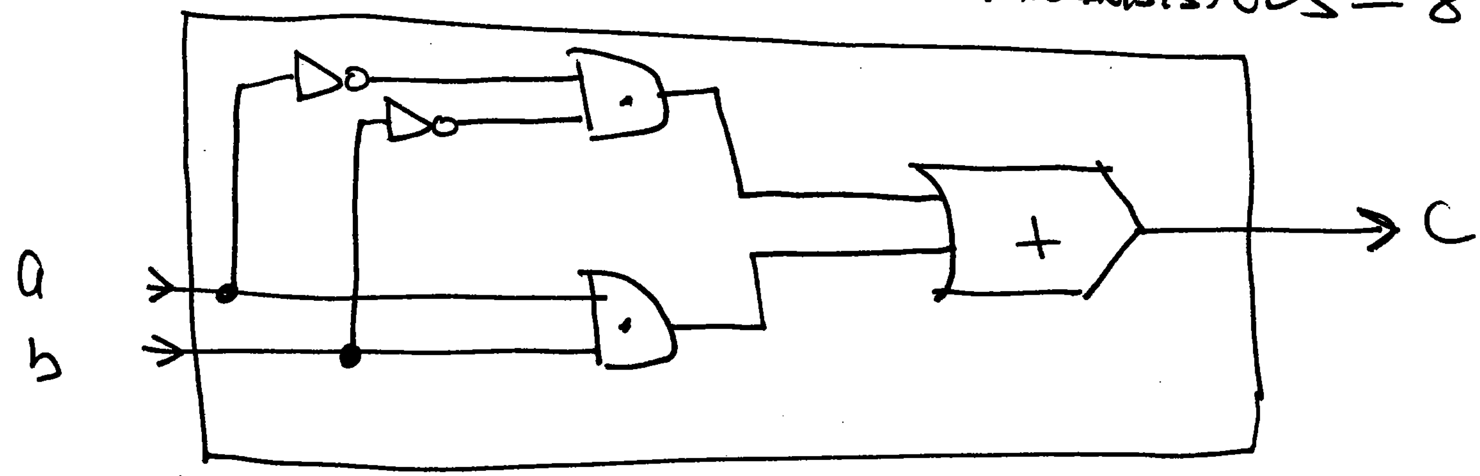


a	b	c	$\bar{a}$	$\bar{b}$	$\bar{a} \cdot \bar{b}$	$a \cdot b$	$(\bar{a} \cdot \bar{b}) + (a \cdot b)$
0	0	1	1	1	1	0	1
0	1	0	1	0	0	0	0
1	0	0	0	1	0	0	0
1	1	1	0	0	0	1	1

OBSERVE $c \equiv (\bar{a} \cdot \bar{b}) + (a \cdot b)$ ✓

CIRCUIT

#TRANSISTORS = 8



EX. CONSTRUCT AN n-BIT COMPARE FOR EQUALITY CIRCUIT, i.e. COMPARE TWO BIT STRINGS OF length n

$$\left.\begin{array}{l}[a_{n-1}a_{n-2}\cdots a_1 a_0]_2 \\ \\ [b_{n-1}b_{n-2}\cdots b_1 b_0]_2\end{array}\right\} \text{OUTPUT} = \begin{cases} 1 & \text{if equal} \\ 0 & \text{if unequal} \end{cases}$$

TRANSISTORS $= 8n + 2(n-1) = 10n - 2$

__GOAL__:

DESIGN AN n-BIT FULL ADDER.

__FIRST__:

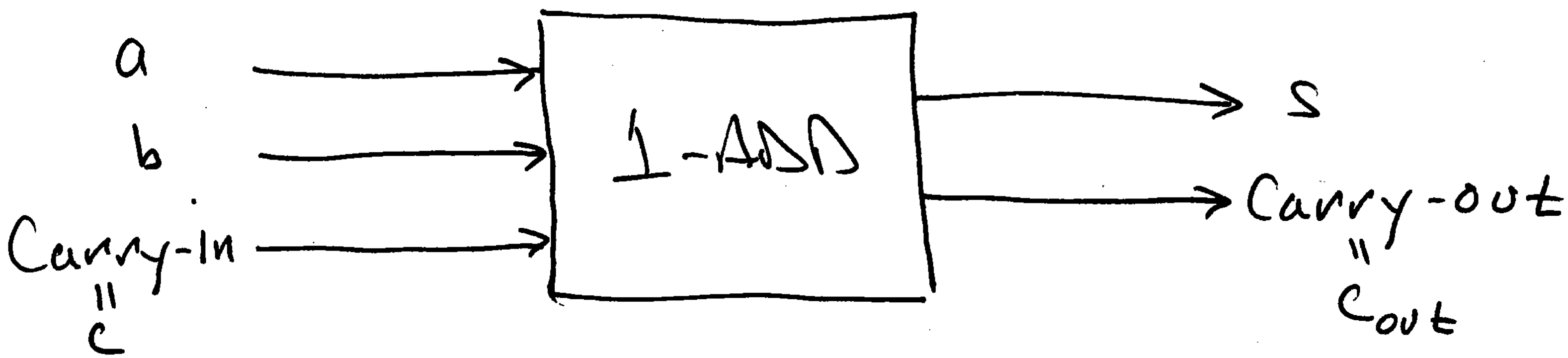
DESIGN A 1-BIT ADDER.

__EX.__

$n=8$

$$
\begin{array}{r}
1\,0\,1\,1\,0\,1\,1\,0\,0 \\
1\,0\,0\,1\,0\,1\,1\,0 \\
1\,0\,1\,1\,0\,0\,1\,1 \\
\hline
1\,0\,1\,0\,0\,1\,0\,0\,1
\end{array}
$$

1-BIT ADDER



a	b	c	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

TO FORM 'SUM-OF-PRODUCTS' EXPRESSIONS

1.) for each output column

2.) for each row CONTAINING A 1 IN THAT COL.

3.) form 'PRODUCT' CORRESPONDING TO INPUTS IN THAT ROW

4.) FORM THE 'SUM' OF THESE 'PRODUCTS'