CMPS 10 2-12-08

THE BASE b POSITIONAL NUMBERING SYSTEM.

WHAT DOES BASE 10 mean?

$$12\,526 = 1\cdot 10^4 + 2\cdot 10^3 + 5\cdot 10^2 + 2\cdot 10^1 + 6\cdot 10^0$$

GIVEN $b > 1$, AN INTEGER, ASSIGN b SYMBOLS TO REPRESENT $0, 1, 2, \ldots, b-1$. THESE SYMBOLS ARE CALLED <u>DIGITS</u>.

A STRING $a_{n-1}a_{n-2}\cdots a_2a_1a_0$ OF n DIGITS STANDS FOR THE INTEGER:

$$[a_{n-1}\cdots a_2a_1a_0]_b = a_{n-1}b^{n-1} + a_{n-2}\cdot b^{n-2} + \cdots + a_2\cdot b^2 + a_1\cdot b^1 + a_0\cdot b^0$$

MORE GENERALLY A FRACTION IS REP. AS:

$$[a_{n-1}\cdots a_1a_0 \bullet a_{-1}a_{-2}\cdots a_{-k}]_b = a_{n-1}b^{n-1} + \cdots + a_0b^0 + a_{-1}b^{-1} + \cdots + a_{-k}b^{-k}$$

base b Point

In CS we are interested in

$b = 2$: binary $\{0, 1\}$

$b = 8$: OCTAL $\{0, 1, 2, 3, 4, 5, 6, 7\}$

$b = 10$: DECIMAL $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$b = 16$: HEXADECIMAL $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$

Ex. $[1011\ 0101\ 1100\ 1]_2$

$= 1 \cdot 2^{12} + 0 \cdot 2^{11} + 1 \cdot 2^{10} + 1 \cdot 2^9 + 0 \cdot 2^8 + 1 \cdot 2^7 + 0 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$

$= 4096 + 1024 + 512 + 128 + 32 + 16 + 8 + 1 = [5817]_{10}$

EX. $[237]_8 = 2 \cdot 8^2 + 3 \cdot 8^1 + 7 \cdot 8^0$

$= 128 + 24 + 7 = [159]_{10}$

EX. $[A17D]_{16} = 10 \cdot 16^3 + 1 \cdot 16^2 + 7 \cdot 16^1 + 13 \cdot 16^0$

$= 41341$

EX. $[357]_{10} = [\ ?\ ]_2$

K	0	1	2	3	4	5	6	7	8	9
2^K	1	2	4	8	16	32	64	128	256	512

$357 = 256 + 101 = 256 + 64 + 37 = 256 + 64 + 32 + 5$

$= 256 + 64 + 32 + 4 + 1 = 1 \cdot 2^8 + 0 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$

$= [101100101]_2$

Count in base 2

2	10
0	= 0
1	= 1
10	= 2
11	
100	= 4
101	
110	
111	
1000	= 8
1001	
1010	
1011	
1100	
1101	
1110	
1111	
10000	= 16

$$[1\underbrace{0\ldots0}_{n}]_2 = 2^n$$

$$[1\underbrace{0\ldots0}_{n}]_b = b^n$$

EXERCISE Count to 100 in base 2.

Ex. $[12.75]_{10} = [\ ?\ ]_2$

$$12.75 = 8 + 4 + .5 + .25 = 2^3 + 2^2 + 2^{-1} + 2^{-2}$$

$$= 1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 0\cdot 2^0 + 1\cdot 2^{-1} + 1\cdot 2^{-2}$$

$$= [1\ 1\ 0\ 0.\ 1\ 1]_2$$

↑ binary Point

Ex. $15.375 = 8 + 4 + 2 + 1 + .25 + .125$ ($.25 = 1/4$, $.125 = 1/8$)

$$= 1\cdot 2^3 + 1\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0 + 0\cdot 2^{-1} + 1\cdot 2^{-2} + 1\cdot 2^{-3}$$

$$= [1111.011]_2$$

Ex. 5438

k	0	1	2	3	4	5	6	7	8	9	10	11	12
2^k	1	2	4	8	16	32	64	128	256	512	1024	2048	4096

$5438 = 4096 + 1342 = 4096 + 1024 + 318$

$= 4096 + 1024 + 256 + 62 = 4096 + 1024 + 256 + 32 + 30$

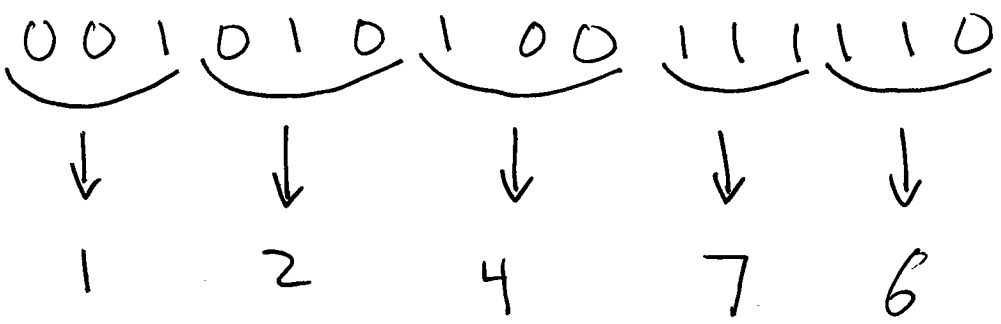
$= 4096 + 1024 + 256 + 32 + 16 + 14$

$= 4096 + 1024 + 256 + 32 + 16 + 8 + 6$

$= 4096 + 1024 + 256 + 32 + 16 + 8 + 4 + 2$

$= [1\,0\,1\,0\,1\,0\,0\,1\,1\,1\,1\,1\,0]_2$

$$[5438]_{10} = [1010100111110]_2$$
$$= [12476]_8$$

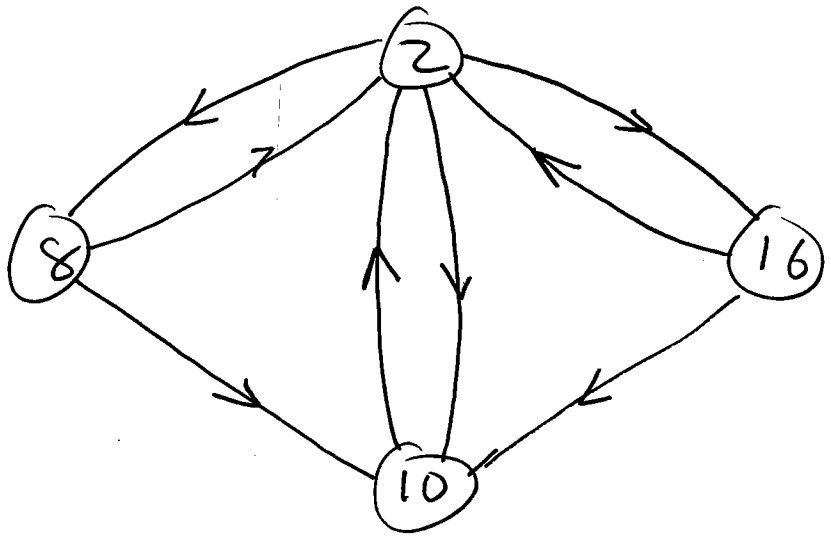


000 = 0
001 = 1
010 = 2
011 = 3
100 = 4
101 = 5
110 = 6
111 = 7

$[5438]_{10} = [153E]_{16}$

0001 0101 0011 1110

1 5 3 E



0000 = 0
0001 = 1
0010 = 2
0011 = 3
0100 = 4
0101 = 5
0110 = 6
0111 = 7
1000 = 8
1001 = 9
1010 = 10 = A
1011 = 11 = B
1100 = 12 = C
1101 = 13 = D
1110 = 14 = E
1111 = 15 = F

THERE is ALWAYS A FIXED # OF BIT AVAILABLE FOR INTEGER STORAGE.

TYPICALLY: 16, 24, 32, OR 64 bits.

EX. IF 16 BITS ARE USED FOR REP. OF POS. INTS., THEN THE LARGEST SUCH INT IS

1111 1111 1111 1111 = 1 0000 0000 0000 0000 − 1

$= 2^{16} - 1 = 65535$

THE SMALLEST IS

0000 0000 0000 0000 = 0

∴ THE RANGE IS 0 TO $2^{16} - 1 = 65535$

THE # OF #S REPRESENTABLE IN THIS WAY IS : 2^{16}

NOTE: THERE ARE 2^n DISTINCT BIT STRINGS OF LENGTH n.

EX. SUPPOSE 64 BITS AVAILABLE.

RANGE : 0 TO $2^{64} - 1$

OF #S : 2^{64}

THUS FAR WE'VE TALKED ABOUT STORING UNSIGNED INTEGERS ONLY. HOW DO WE STORE SIGNED INTEGERS?

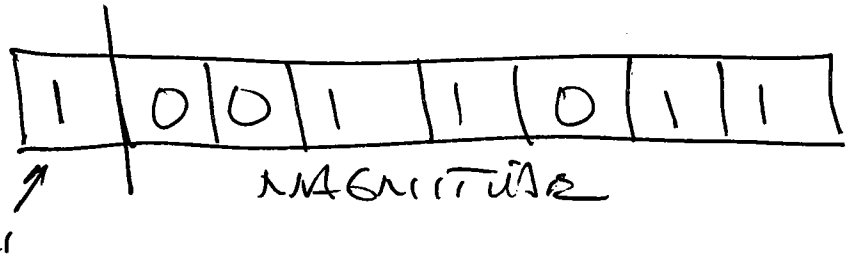
- USE A SIGN BIT { 0 FOR +, 1 FOR − }

SIGN MAGNITUDE REPRESENTATION

- LEFTMOST BIT IS THE SIGN BIT
- REMAINING BITS GIVE THE MAGNITUDE.

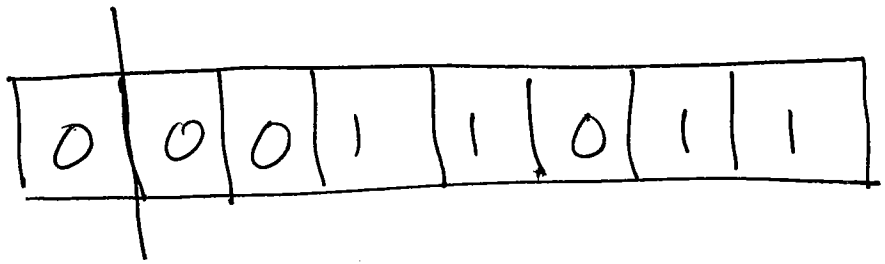
EX. SUPPOSE 8 BITS ARE AVAILABLE FOR SIGN/MAGNITUDE REP. OF INTEGERS.

$-27 =$



$27 = 16+11 = 16+8+2+1 = 11011 = 0011011$

$+27 =$



HOW DOES COMPUTER KNOW THAT 10011011 REPRESENTS -27 AND NOT 155 ? ANSWER: IT DOESN'T KNOW.

RANGE OF #s :

$1\ 1111\ 111 = -(2^7 - 1) = -127$

TO

$0\ 1111\ 111 = +(2^7 - 1) = +127$