CMPS 10 11-4-10

Logic operators:	Symbols: math	Symbols: circuits
conjunction (and)	$\wedge$	$\cdot$
disjunction (or)	$\vee$	$+$
Negation (not)	$\neg$	$\overline{exp}$
exclusive or (exor)	$\overline{\vee}$	$\oplus$

Logical constants		
false	F	0
true	T	1

Truth Tables

Negation:

a	$\bar{a}$
0	1
1	0

conjunction, disjunction, exclusive or

a	b	$a \cdot b$	$a + b$	$a \oplus b$
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

Logical Identities:

- commutative: $a \cdot b = b \cdot a$

  $a + b = b + a$

  $a \oplus b = b \oplus a$

- associative: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

  $a + (b + c) = (a + b) + c$

  $a \oplus (b \oplus c) = (a \oplus b) \oplus c$

- Distributive: $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$

  $a + (b \cdot c) = (a + b) \cdot (a + c)$

- Demorgan: $\overline{a \cdot b} = \bar{a} + \bar{b}$

  $\overline{a + b} = \bar{a} \cdot \bar{b}$

- other: $a \cdot 1 = a$ $\quad a \cdot 0 = 0$

  $a + 1 = 1$ $\quad a + 0 = a$

Truth table for 1st DeMorgan:

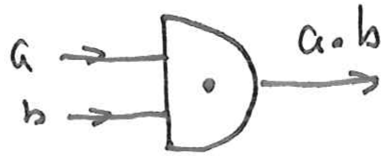
$\overline{a \cdot b} = \bar{a} + \bar{b}$ ✓

a	b	$a \cdot b$	$\overline{a \cdot b}$	$\bar{a}$	$\bar{b}$	$\bar{a} + \bar{b}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

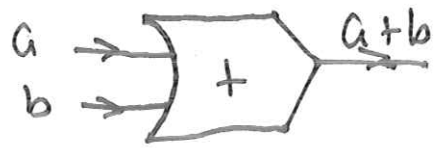
1st Distributive: $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ ✓

a	b	c	$b + c$	$a \cdot (b+c)$	$a \cdot b$	$a \cdot c$	$(a \cdot b) + (a \cdot c)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

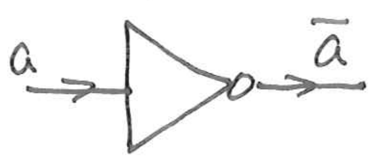
Logic Gates:

and: 

a	b	$a \cdot b$
0	0	0
0	1	0
1	0	0
1	1	1

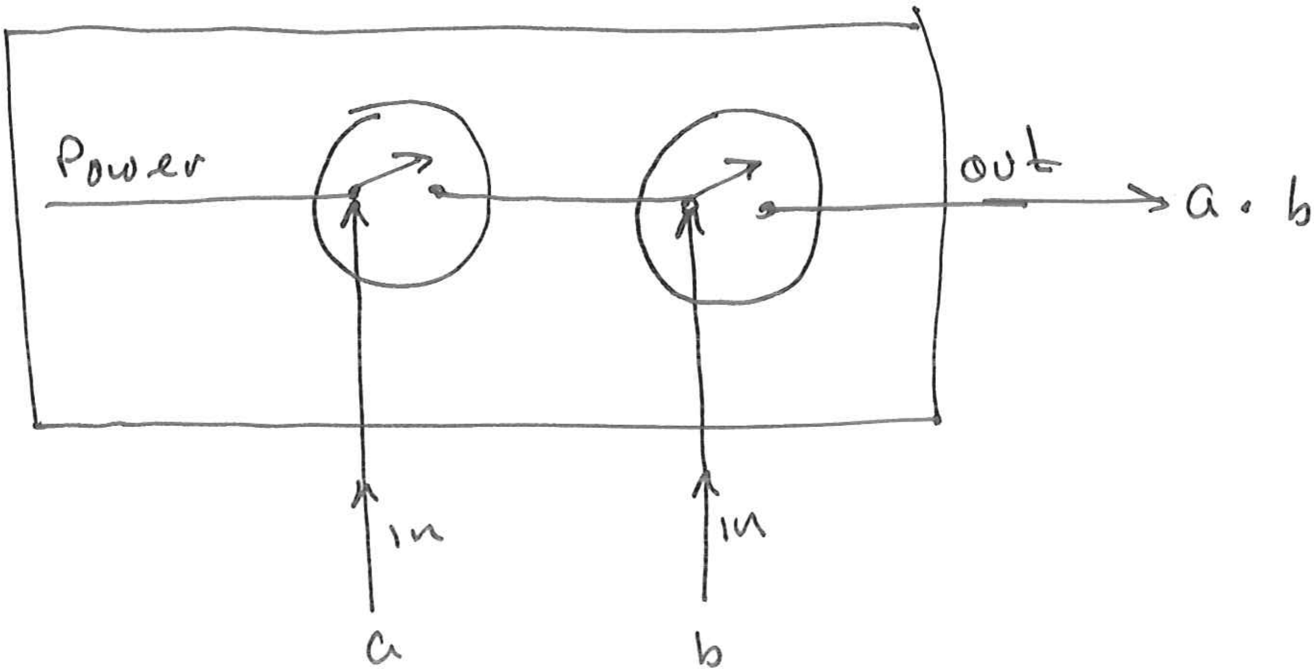
or: 

a	b	$a + b$
0	0	0
0	1	1
1	0	1
1	1	1

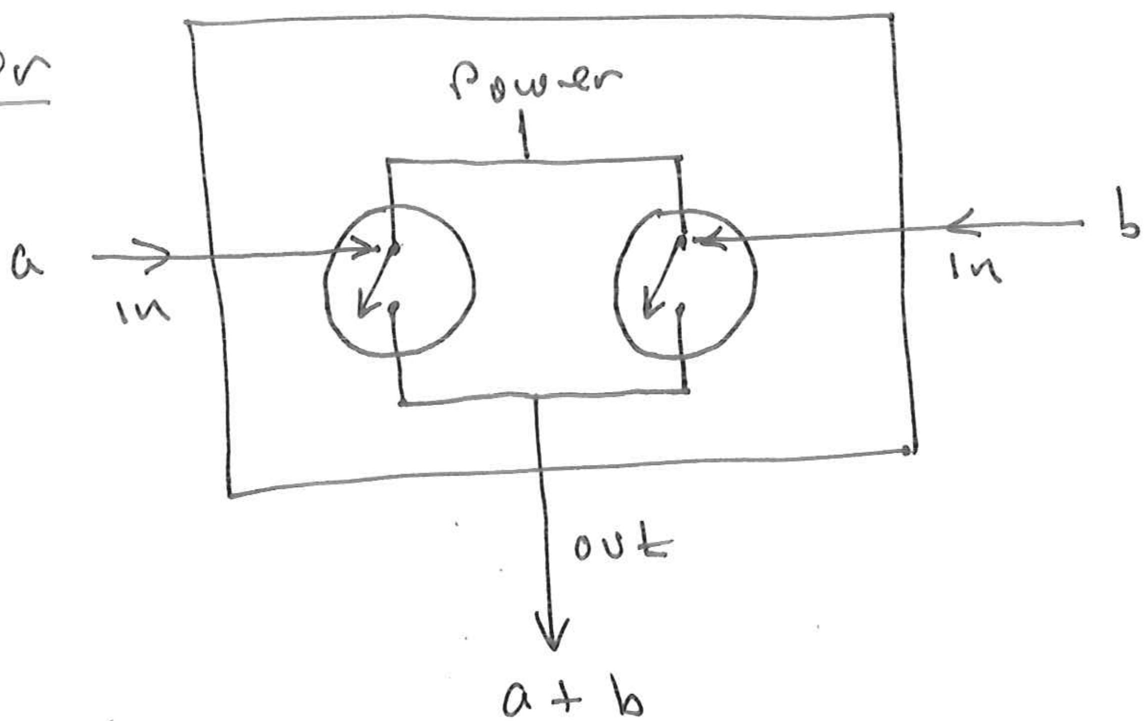
not: 

a	$\bar{a}$
0	1
1	0

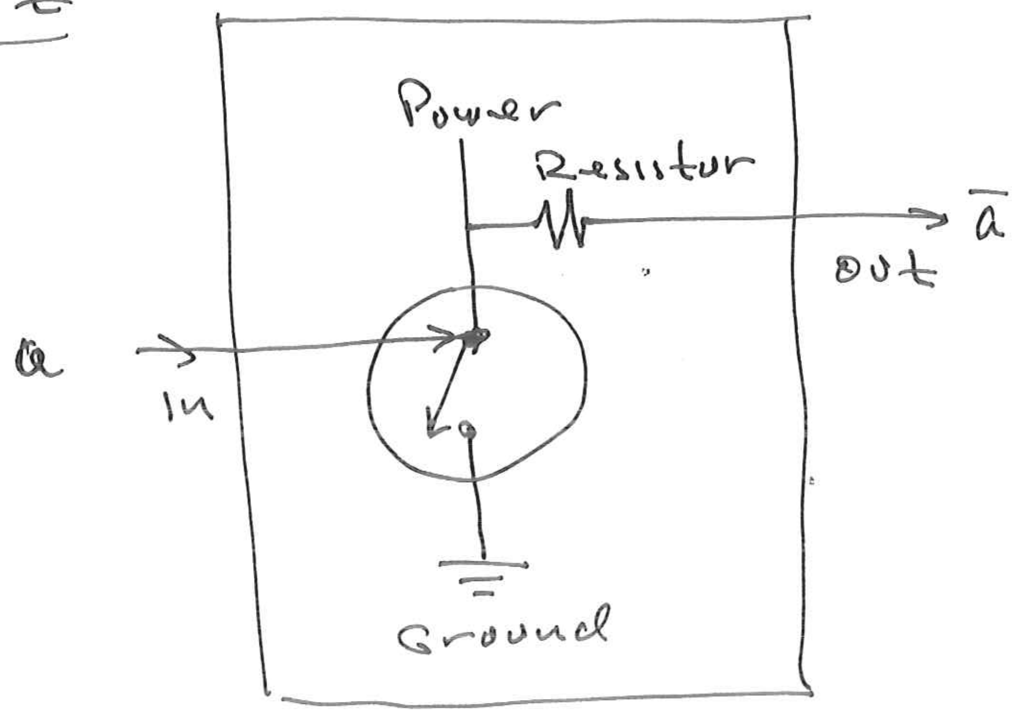
and



Or



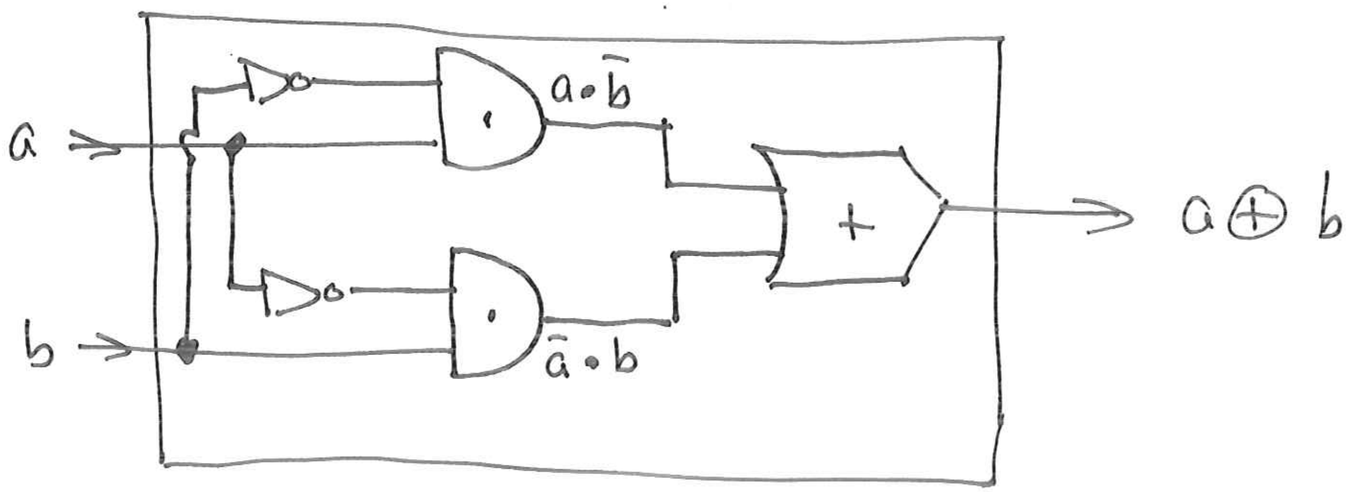
not



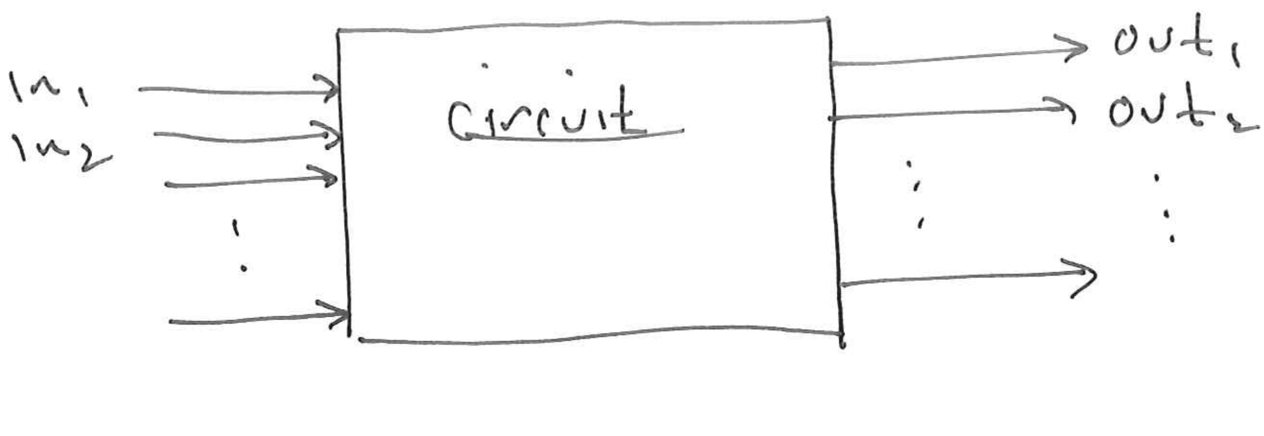
another identity: $a \oplus b = (a \cdot \bar{b}) + (\bar{a} \cdot b)$

a	b	$a \oplus b$	$\bar{a}$	$\bar{b}$	$a \cdot \bar{b}$	$\bar{a} \cdot b$	$(a \cdot \bar{b}) + (\bar{a} \cdot b)$
0	0	0	1	1	0	0	0
0	1	1	1	0	0	1	1
1	0	1	0	1	1	0	1
1	1	0	0	0	0	0	0

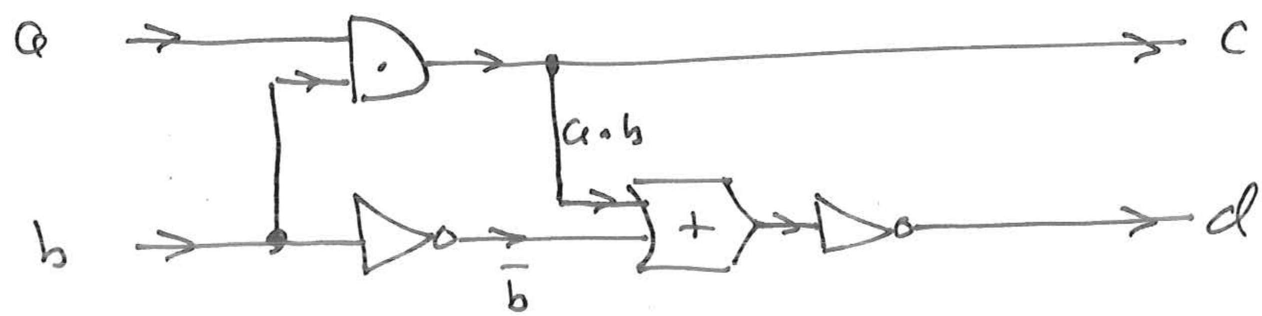
exor gate:



a <u>Combinational circuit</u> is a collectio of logic gates that transforms some binary inputs into binary outputs



Ex. #Transistors = 2 + 1 + 2 + 1 = 6



a	b	c	d
0	0	0	0
0	1	0	1
1	0	0	0
1	1	1	0

note: $c = a \cdot b$

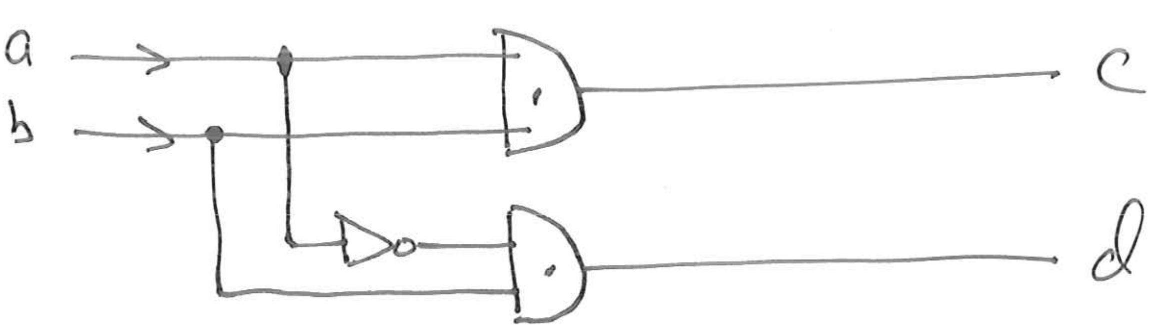
$d = \overline{(a \cdot b) + \bar{b}} = \bar{a} \cdot b$

Exercise: show these are equivalent by truth table

a	b	$\bar{a}$	$\bar{b}$	$\bar{a}\cdot b$	$a\cdot b$	$(a\cdot b)+\bar{b}$	$\overline{(a\cdot b)+\bar{b}}$
0	0	1	1	0	0	1	0
0	1	1	0	1	0	0	1
1	0	0	1	0	0	1	0
1	1	0	0	0	1	1	0

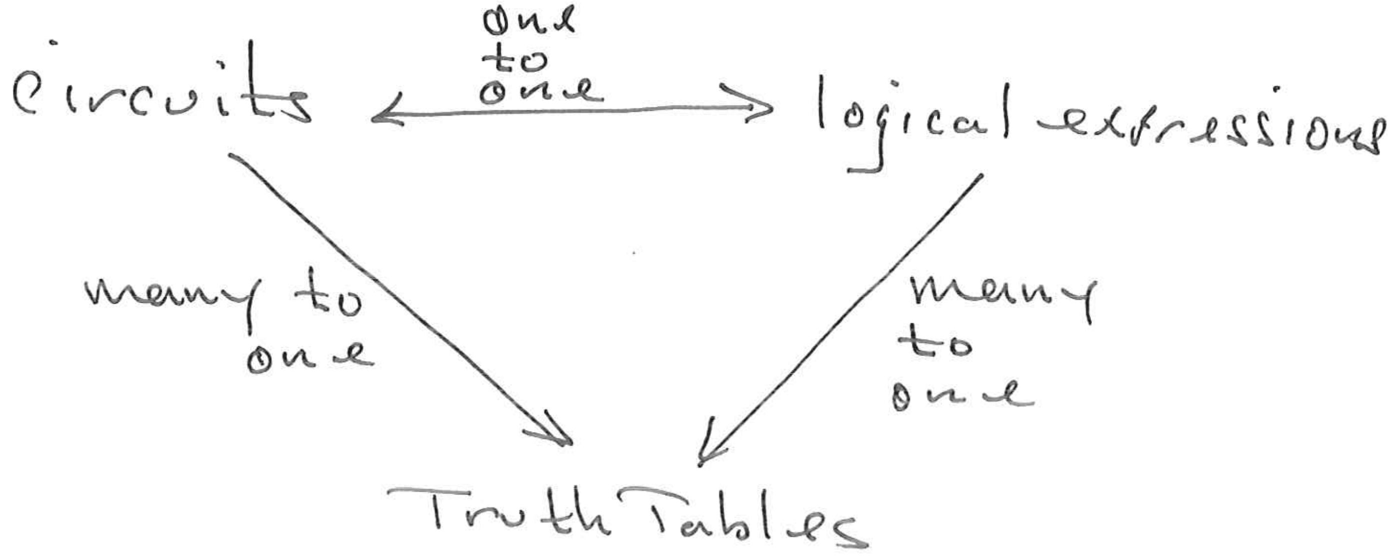
now that: $c = a\cdot b$

$d = \bar{a}\cdot b$



$\#$ transistors $= 2 + 2 + 1 = 5$

Correspondences:

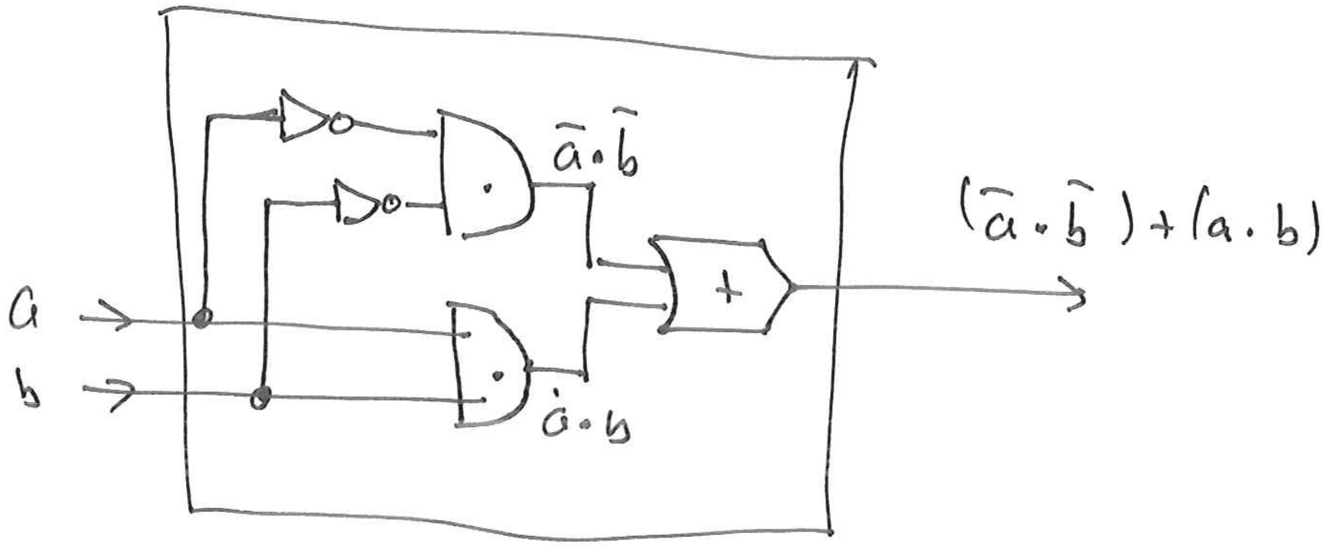


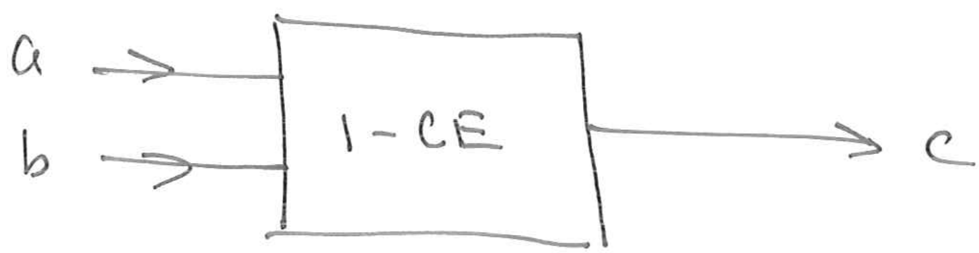
Ex. design a 1-Bit Compare for equality circuit

a	b	c
0	0	1
0	1	0
1	0	0
1	1	1

note: $c = (\bar{a} \cdot \bar{b}) + (a \cdot b)$

Exercise: check this





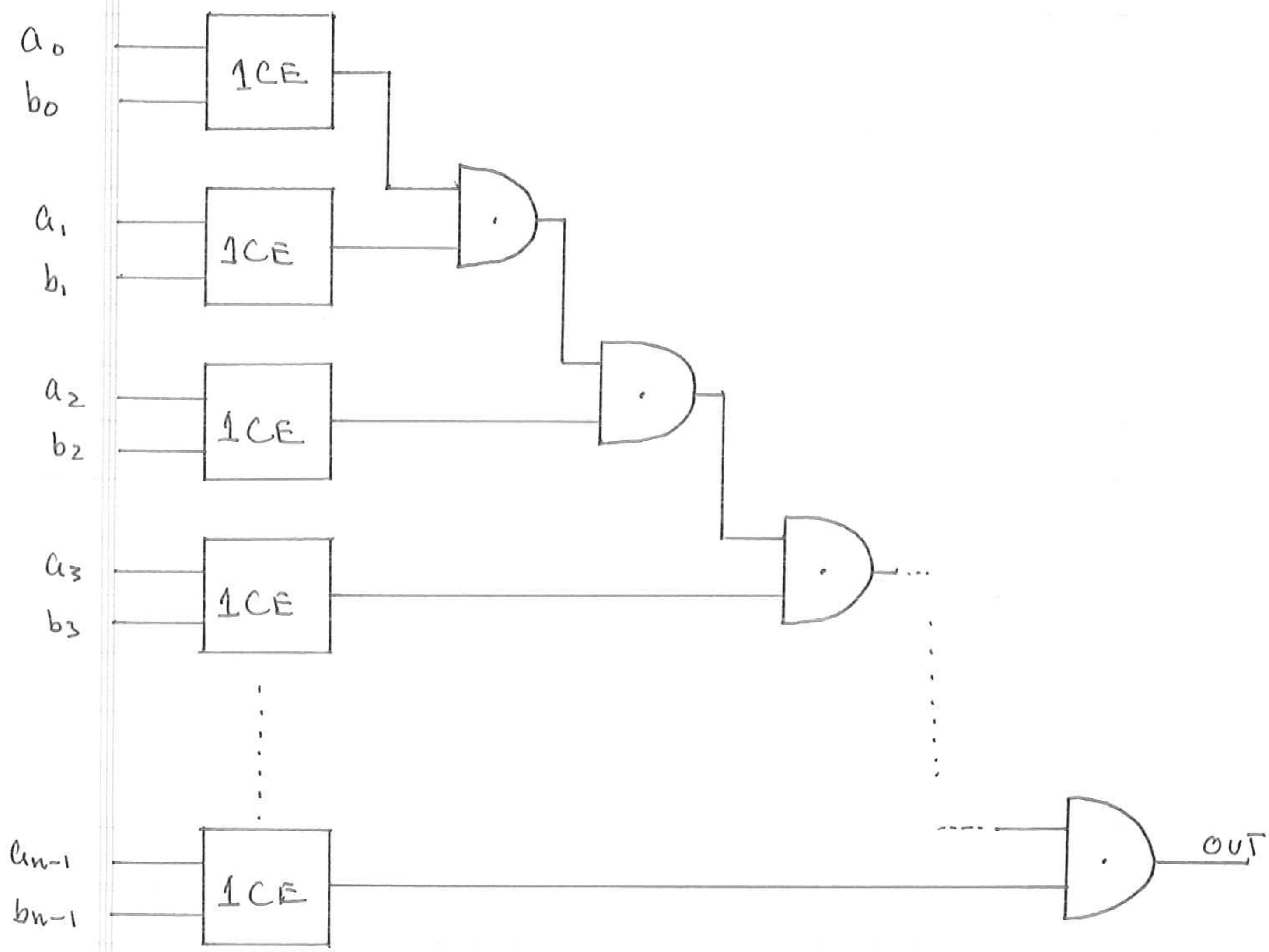
<u>Ex.</u> n-bit comp for equality

<u>Inputs</u>: $a_{n-1}\, a_{n-2} \cdots a_1\, a_0$

$b_{n-1}\, b_{n-2} \cdots b_1\, b_0$

<u>Output</u>: 1 if equal 0 otherwise

INPUTS: TWO n BIT BINARY NUMBERS
$[a_{n-1} \cdots a_0]_2$, $[b_{n-1} \dots b_0]_2$
OUTPUT: 1 IF EACH $a_i = b_i$ $(0 \le i \le n-1)$,
0 OTHERWISE



HOW MANY ROWS AND COLUMNS WOULD THE TRUTH TABLE FOR THIS CIRCUIT HAVE ?

ANSWER: 2^{2n} ROWS
$2n+1$ COLUMNS