CMPS 10 10-28-10

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chap. 4

Base b Positional numbering System

Base 10:

Ex. what does 12526 mean?

$$[12526]_{10} = 1\cdot 10^{4} + 2\cdot 10^{3} + 5\cdot 10^{2} + 2\cdot 10^{1} + 6\cdot 10^{0}$$

$$[35.62]_{10} = 3\cdot 10^{1} + 5\cdot 10^{0} + 6\cdot 10^{-1} + 2\cdot 10^{-2}$$

Base b!

Given $b > 1$ and symbols called digits assigned to $\{0, 1, 2, \ldots, b-1\}$,

a string $a_{n-1}a_{n-2}\cdots a_1 a_0$ of digits

means:

$$[a_{n-1}\cdots a_0]_b = a_{n-1}\cdot b^{n-1} + a_{n-2}\cdot b^{n-2} + \cdots + a_2\cdot b^2 + a_1\cdot b^1 + a_0\cdot b^0$$

also

$$[a_{n-1}\cdots a_0 . a_{-1}a_{-2}\cdots a_{-K}]_b$$

$$= a_{n-1}\cdot b^{n-1} + \cdots + a_0\cdot b^0 + a_{-1}\cdot b^{-1} + \cdots + a_{-K}\cdot b^{-K}$$

$$[10]_b = 1\cdot b^1 + 0\cdot b^0 = b$$

most interested in.

$b=2$: Binary $\{0,1\}$

$b=8$: Octal $\{0,1,2,3,4,5,6,7\}$

$b=16$: Hexidecimal $\{0,1,\ldots,9,A,B,C,D,E,F\}$

$b=10$: Decimal $\{0,\ldots,9\}$

Ex. Count in base 2:

0 = 0
1 = 1
10 = 2
11
100
101
110
111
1000
1001
1010 = 10
1011 = 11

1100 = 12
1101 = 13
1110
1111 = 15
10000
.
.
.

Ex $[1111]_2 = 1\cdot 2^3 + 1\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0$

$= 8 + 4 + 2 + 1$

$= 12 + 3 = [15]_{10}$

Ex. $[1011010111001]_2$

$= 1\cdot 2^{12} + \cancel{0\cdot 2^{11}} + 1\cdot 2^{10} + 1\cdot 2^9 + \cancel{0\cdot 2^8} + 1\cdot 2^7 + \cancel{0\cdot 2^6}$
$+ 1\cdot 2^5 + 1\cdot 2^4 + 1\cdot 2^3 + \cancel{0\cdot 2^2} + \cancel{0\cdot 2^1} + 1\cdot 2^0$

$= 2^{12} + 2^{10} + 2^9 + 2^7 + 2^5 + 2^4 + 2^3 + 1$

$= 4096 + 1024 + 512 + 128 + 32 + 16 + 8 + 1$

$= [5817]_{10}$

Ex. $[237]_8 = 2 \cdot 8^2 + 3 \cdot 8 + 7$

$= 2 \cdot 64 + 24 + 7$

$= [159]_{10}$

Ex. $[A17D]_{16} = 10 \cdot 16^3 + 1 \cdot 16^2 + 7 \cdot 16 + 13$

$= [41341]_{10}$

Ex. $[357]_{10} = [\ ?\ ]_2$

k	0	1	2	3	4	5	6	7	8	9
2^k	1	2	4	8	16	32	64	128	256	512

$357 = 256 + 101$

$= 256 + 64 + 37$

$= 256 + 64 + 32 + 5$

$= 256 + 64 + 32 + 4 + 1$

$$[357]_{10} = 2^8 + 2^6 + 2^5 + 2^2 + 2^0$$

$$= 1\cdot 2^8 + 0\cdot 2^7 + 1\cdot 2^6 + 1\cdot 2^5 + 0\cdot 2^4 + 0\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0$$

$$= [1\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 1]_2$$

Ex. $[\underbrace{101}_{5},\underbrace{100}_{4},\underbrace{101}_{5}]_2 = [\ ?\ ]_8$

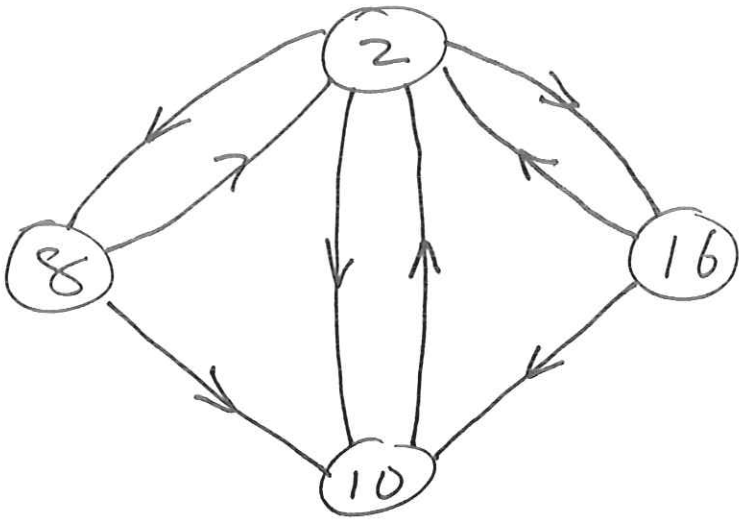
Oct	Bin
0 =	000
1 =	001
2 =	010
3 =	011
4 =	100
5 =	101
6 =	110
7 =	111

$$= [5\ 4\ 5]_8$$

Ex. $[00{\cdot}0\underbrace{1}_{1}0\underbrace{1}_{6}1\underbrace{0}_{5}0\underbrace{1}_{5}01]_2 = [\ 165\ ]_{16}$

Hex	Bin
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Conversions



Ex.

$$[12.75]_{10} = [\ ?\ ]_2$$

$$12.75 = 8 + 4 + .5 + .25$$

$$= 8 + 4 + \frac{1}{2} + \frac{1}{4}$$

$$= 2^3 + 2^2 + 2^{-1} + 2^{-2}$$

$$= [1100.11]_2$$

Ex. $15.375 = 8 + 4 + 2 + 1 + .25 + .125$

$$= 2^3 + 2^2 + 2^1 + 2^0 + 2^{-2} + 2^{-3}$$

$$= [1111.011]_2$$

Ex. Addition

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1\ 1\ 1\ \ \ 1} \\ 1\,0\,1\,1\,0\,0\,1 \\ +\ 1\,1\,0\,1\,0\,0\,1 \\ \hline 1\,1\,0\,0\,0\,0\,1\,0 \end{array}$$

Ex. $\begin{array}{r} {\scriptstyle 1\ \ 1\ 1} \\ [545]_8 \\ +\ [763]_8 \\ \hline [1530]_8 \end{array}$

Ex.

If 16 bits are available for (unsigned) integer representation, Then

Smallest = 0000 0000 0000 0000 = 0

largest = 1111 1111 1111 1111 =

= 1 0000 0000 0000 0000 - 1

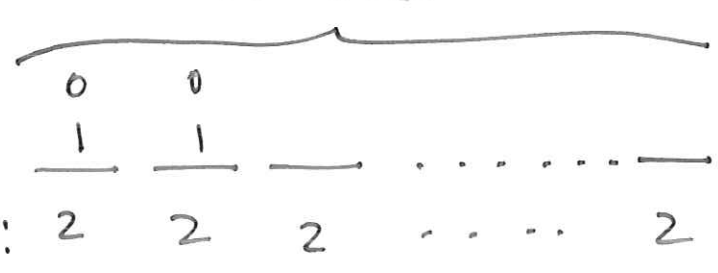
= $2^{16} - 1$

= 65536 - 1

= 65535

of #s = 2^{16}

(# of bit strings of length n) $= 2^n$



total # choices $= \underbrace{2 \cdot 2 \cdots \cdot 2}_{n \text{ factors}} = 2^n$

Ex. Same question except 32 bits

smallest = 0

largest $= 2^{32} - 1 = 4,294,967,295$

of #s $= 2^{32}$

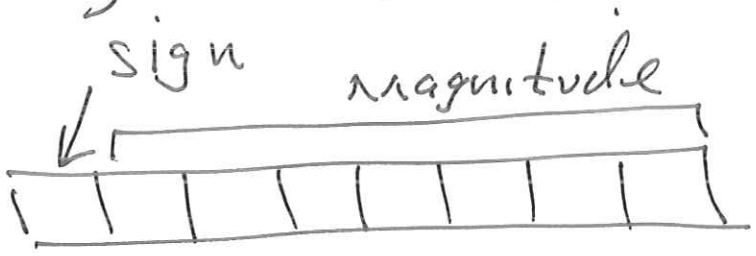
Signed Integers

Sign Magnitude Representation:

leftmost is sign bit: 0 = +, 1 = −

remaining bits are magnitude:

Ex. Suppose 8 bits are used for sign magnitude Rep



$-27 = 1\ 0011011$

$+27 = 0\ 0011011$