CMPS 10 10-21-10

Recall: Informal Defn.

$f(n) = \Theta(g(n))$ if

$f(n) = \text{const} \cdot g(n) + h(n)$ ← lower order terms

where $\frac{h(n)}{g(n)} \to 0$ as $n \to \infty$

Equivalently

$\frac{f(n)}{g(n)} \to \text{const.}$ as $n \to \infty$ (const: Positive real no.)

Ex. $2n^2 + 3n + 5 = \Theta(n^2)$

Ex. $10 \cdot n^{7/3} + n^{5/2} + 1 = \Theta(n^{5/2})$

Ex. $an^2 + bn + c = \Theta(n^2)$

The Asymptotic Run Times of The Sorting Algorithms.

	Best	Avg	Worst
Selection	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$
Bubble	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$
Insertion	$\Theta(n)$	$\Theta(n^2)$	$\Theta(n^2)$

Consider 4 Algorithms A, B, C, D (solving the same Problem) with worst case costs

	# Basic ops	Asymp. Run Time
A:	$1000n$	$\Theta(n)$
B:	$10n^2$	$\Theta(n^2)$
C:	n^2	$\Theta(n^2)$
D:	$n^2+100n+1000$	$\Theta(n^2)$

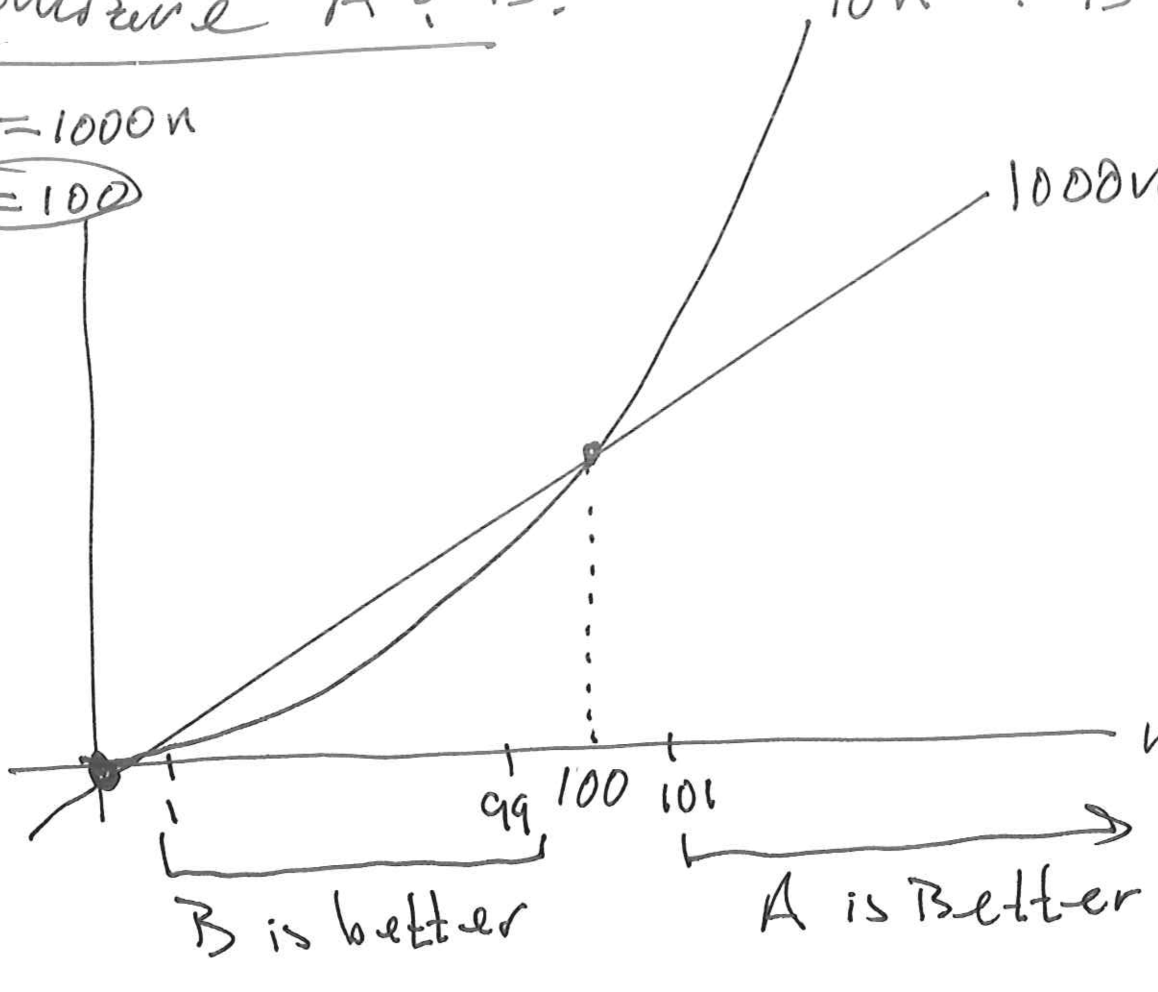
Compare C & D: The lower order terms 100n, 1000 are negligible compared to n^2 for large n.

Compare B & C: we can equalize these by running B on a faster machine.

Compare A & B:

$10n^2 = 1000n$

$n = 100$



Ex.

A: $12n\sqrt{n}$ Basic ops $\Theta(n^{3/2})$

B: $3n^2$ Basic ops $\Theta(n^2)$

Find Crossover pt.

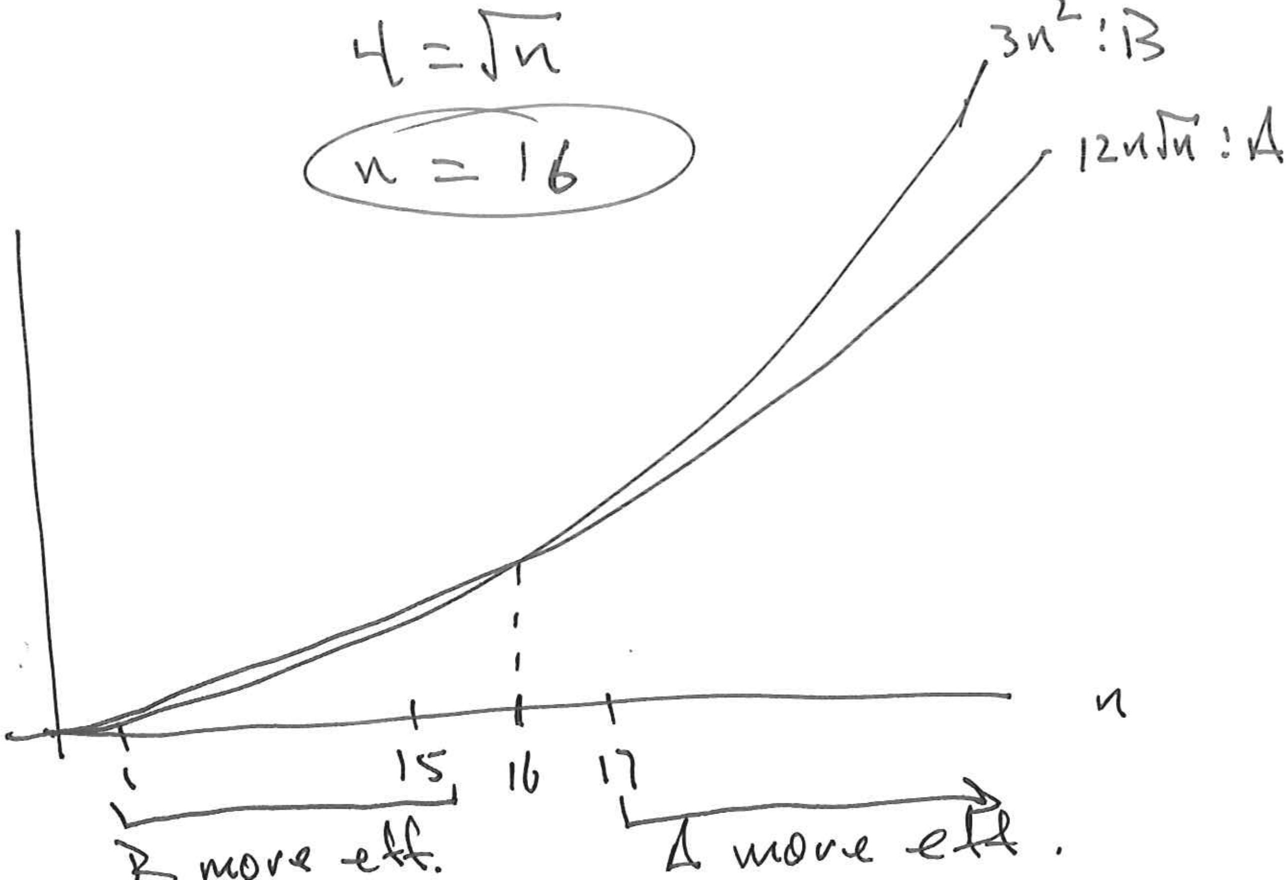
$$12n\sqrt{n} = 3n^2$$

$$4\cancel{n}\sqrt{n} = \cancel{n}\cdot n$$

$$4\cancel{\sqrt{n}} = n = \cancel{\sqrt{n}}\cdot\sqrt{n}$$

$$4 = \sqrt{n}$$

$$n = 16$$



Binary Search

As before

Input: $n \geq 1$

$a_1 \dots a_n$ } must be sorted

target

Output: the index i for which $a_i ==$ target or 0 if no such i exists.

Ex. n = 10   target = 11

| 2 3 4 7 (10) 11 15 20 25 28 |
L ... R

2 3 4 7 10 | 11 15 (20) 25 28 |
L ... R

2 3 4 7 10 | (11) 15 | 20 25 28
L ... R

return 6

comparisons = 3

Ex. Same list, target 3

| 2 3 4 7 (10) 11 15 20 25 28 |

L R

| 2 (3) 4 7 | 10 11 15 20 25 28

L R

return 2

comp = 2

Ex. Same list, target = 8

| 2 3 4 7 (10) 11 15 20 25 28 |
L R

| 2 (3) 4 7 | 10 11 15 20 25 28
L R

2 3 | (4) 7 | 10 11 15 20 25 28
L R

2 3 4 | (7) | 10 11 15 20 25 28
L R

2 3 4 7 | | 10 11 15 20 25 28
R L

return 0

comp = 4

```
if cond1
    stmt
else
    if cond2
        stmt
    else
        if cond3
            stmt
        else
            stmt
```

```
if cond1
    stmt
else if cond2
    stmt
else if cond3
    stmt
else
    stmt
```

Binary Search

```
L = 1
R = n
found = false
while L ≤ R and not found
    m = ⌊(L+R)/2⌋
    if target == a_m
        found = true
    else if target < a_m
        R = m-1
    else
        L = m+1
if not found
    m = 0
Print m
stop
```

Defn

Let x be a real number. The floor of x is:

$\lfloor x \rfloor$ = greatest integer less than or equal to x

(i.e. the integer part of x)

The ceiling of x is:

$\lceil x \rceil$ = least integer greater than or equal to x.