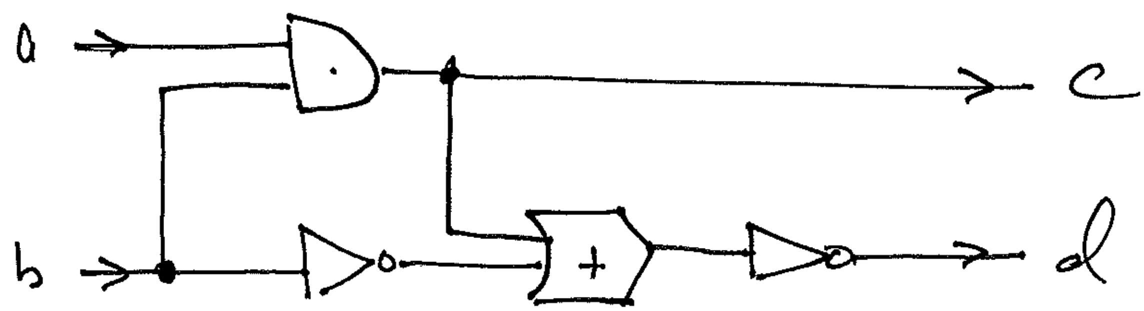
CMPS 10 11-12-09

Recall:




6 TRANSISTORS.

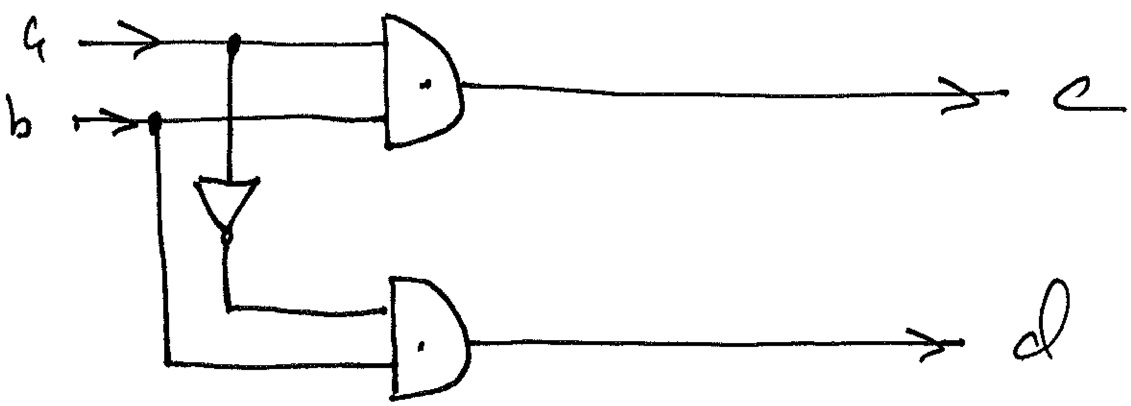
Truth Table:

| a | b | c | d |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 |

note: $c \equiv a \cdot b$

$d \equiv \bar{a} \cdot b$

An Equivalent circuit




5 Transistors

NOTE: from 1st circuit:

$$d \equiv \overline{\bar{b} + (a \cdot b)}$$

Truth Table showing $\overline{\bar{b} + (a \cdot b)} \equiv \bar{a} \cdot b$ ✓

| $a$ | $b$ | $\bar{a}$ | $\bar{b}$ | $(a \cdot b)$ | $\bar{b} + (a \cdot b)$ | $\overline{\bar{b} + (a \cdot b)}$ | $\bar{a} \cdot b$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |

CORRESPONDANCES:

$$\begin{tikzcd}
\text{CIRCUITS} \arrow[rr, leftrightarrow, "1-1"] \arrow[dr, "\text{many to 1}"'] & & \text{logical expressions} \arrow[dl, "\text{many to 1}"] \\
& \text{TRUTH TABLES} &
\end{tikzcd}$$

Ex. Design a 1-bit compare for equality circuit. i.e. given inputs a, b output 1 if $a = b$, 0 if $a \neq b$.

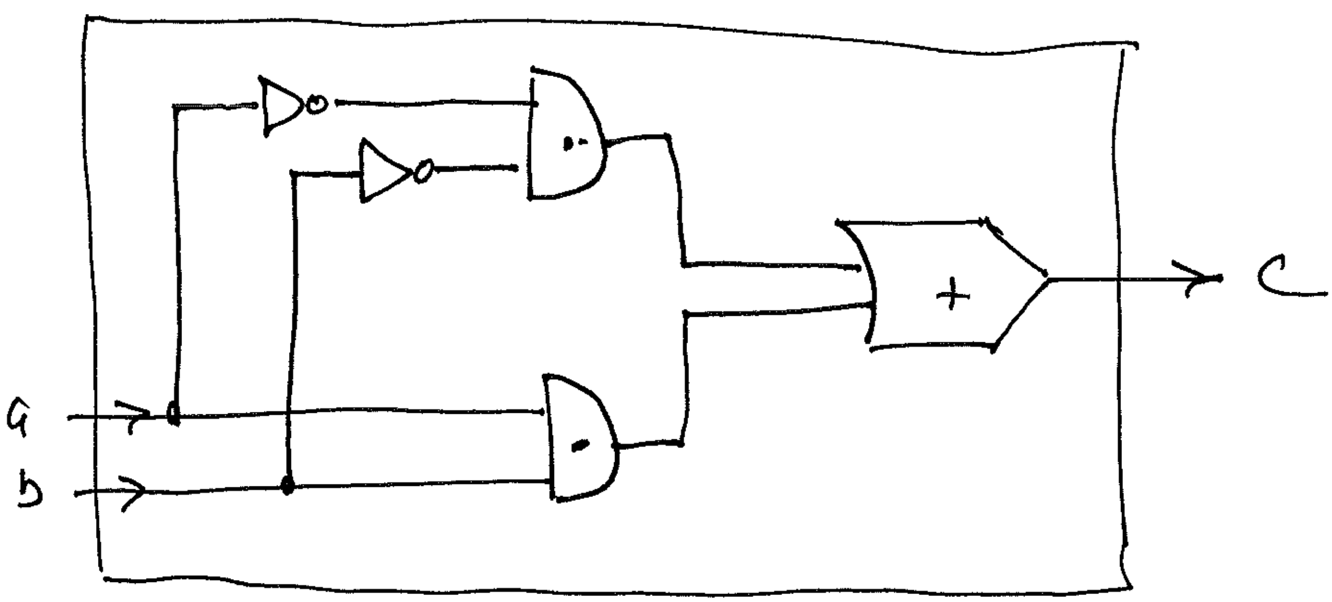
| a | b | c |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

observe! $c \equiv (\bar{a} \cdot \bar{b}) + (a \cdot b)$

check

| $a$ | $b$ | $\bar{a}$ | $\bar{b}$ | $\bar{a} \cdot \bar{b}$ | $a \cdot b$ | $\bar{a} \cdot \bar{b} + a \cdot b$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 |

circuit

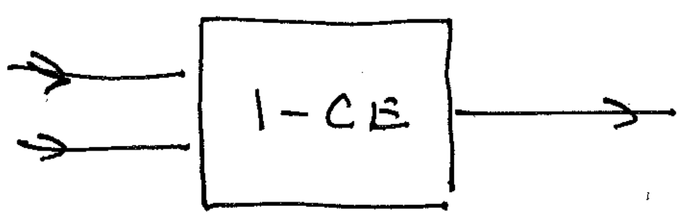



1-CE

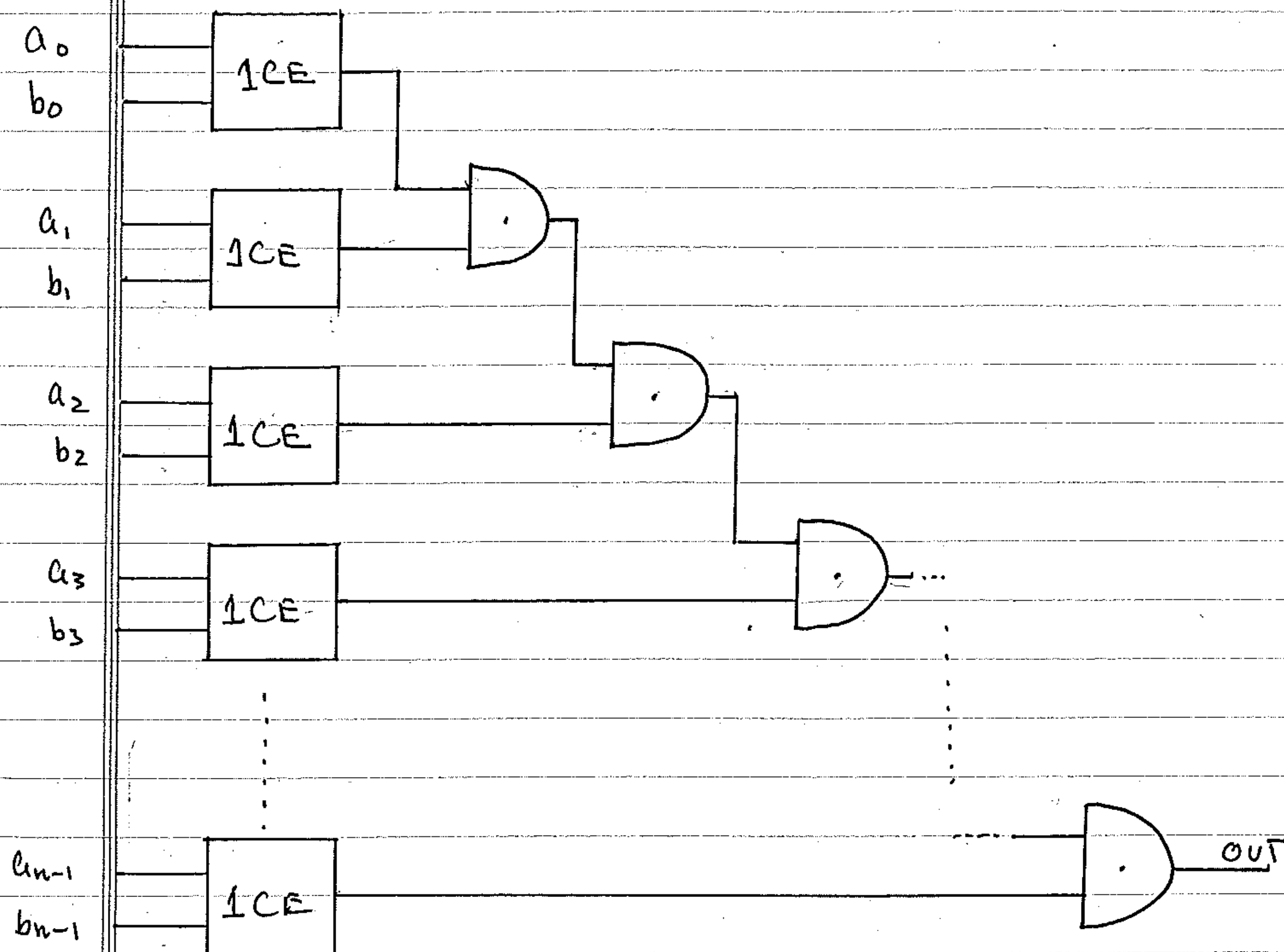
Ex. n-Bit compare for equality circuit. Input: 2 bit strings of length n : $a_0 \cdots a_{n-1}$, $b_0 \cdots b_{n-1}$

Output : 1 if 2 bit strings are identical, 0 otherwise.

Use 1-CE circuit



INPUTS: TWO $n$ BIT BINARY NUMBERS
$[a_{n-1} \cdots a_0]_2$, $[b_{n-1} \ldots b_0]_2$
OUTPUT: 1 IF EACH $a_i = b_i$ $(0 \le i \le n-1)$,
0 OTHERWISE




HOW MANY ROWS AND COLUMNS WOULD THE TRUTH TABLE FOR THIS CIRCUIT HAVE ?

ANSWER: $2^{2n}$ ROWS
$2n+1$ COLUMNS

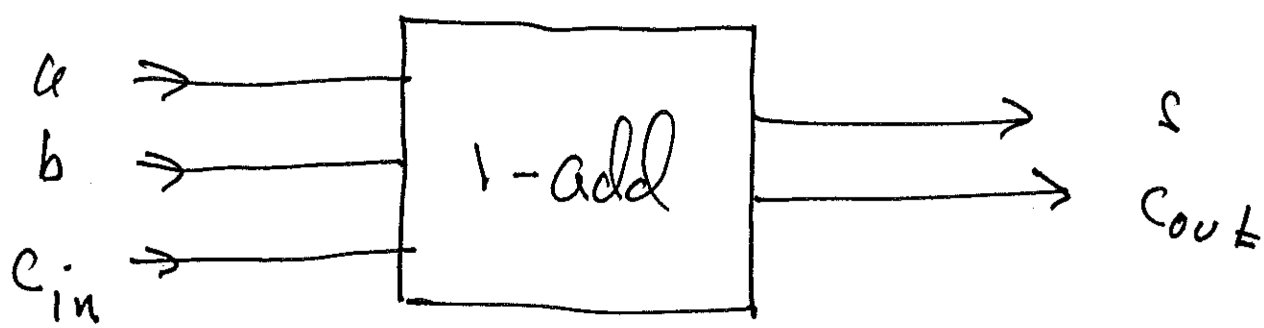
<u>Ex.</u> Goal: Design $n$-bit full adder

<u>ex.</u> addition in binary $\quad n = 8$

$$
\begin{array}{r}
1\,0\,1\,1\,0\,1\,1\,0\,0 \\
1\,0\,0\,1\,0\,1\,1\,0 \\
1\,0\,1\,1\,0\,0\,1\,1 \\
\hline
1\,0\,1\,0\,0\,1\,0\,0\,1
\end{array}
$$

$\begin{cases} \text{Inputs: } a, b, c_{in} \\ \text{Outputs: } s, c_{out} \end{cases}$

First build a 1-bit adder



## Truth Table:

| inputs | | | outputs | |
|---|---|---|---|---|
| a | b | c | $C_{out}$ | S |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

## Circuit Design algorithm

1.) for each output col.

2.) for each row containing a 1 in that col.

3.) form 'product' of corresponding inputs

4.) form 'sum' of these products

5.) Draw circuit corresponding to this set of logical expressions.

from truth table we get

$$C_{out} \equiv (\bar{a}\cdot b\cdot c)+(a\cdot\bar{b}\cdot c)+(a\cdot b\cdot\bar{c})+(a\cdot b\cdot c)$$

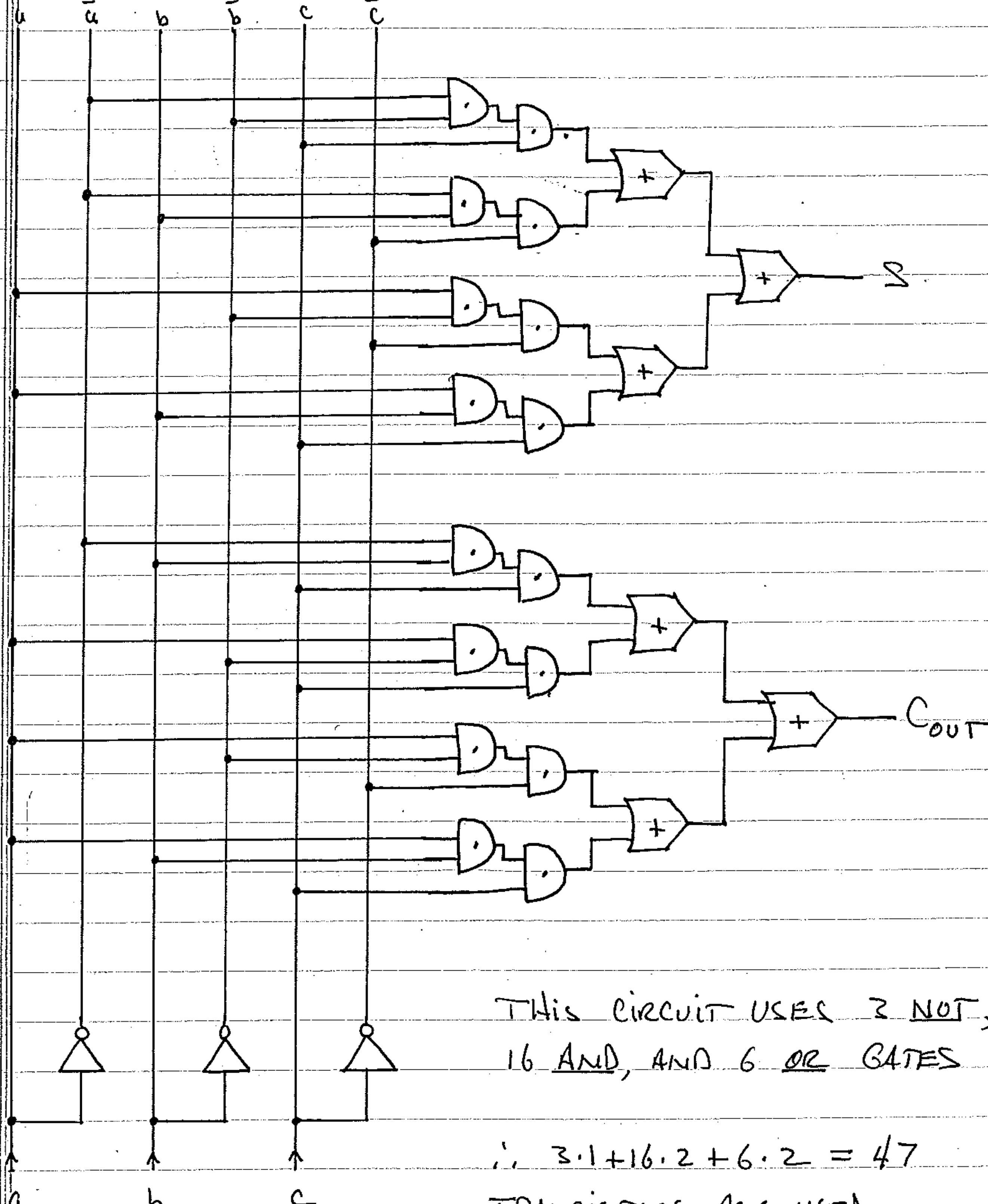
$$S \equiv (\bar{a}\cdot\bar{b}\cdot c)+(\bar{a}\cdot b\cdot\bar{c})+(a\cdot\bar{b}\cdot\bar{c})+(a\cdot b\cdot c)$$

Now Draw circuit !

note

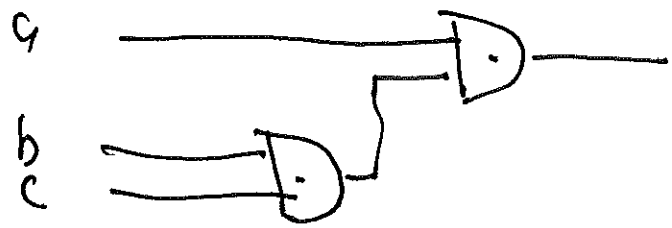
$$S \equiv \left[((\bar{a}\cdot\bar{b})\cdot c)+((\hat{a}\cdot b)\cdot\bar{c})\right]+\left[((a\cdot\bar{b})\cdot\bar{c})+((a\cdot b)\cdot c)\right]$$

WE NEED A SYSTEMATIC WAY TO DRAW CIRCUITS:

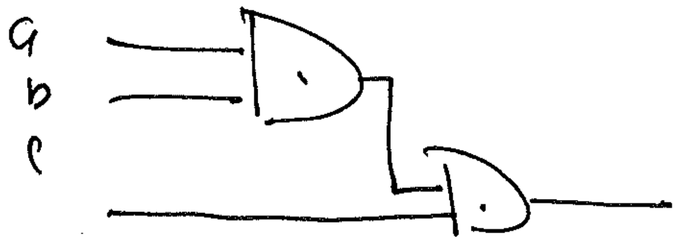



THIS CIRCUIT USES 3 NOT, 16 AND, AND 6 OR GATES

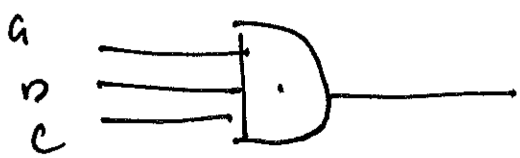
$\therefore$ $3\cdot1+16\cdot2+6\cdot2 = 47$ TRANSISTORS ARE USED.




$a \cdot (b \cdot c)$




$(a \cdot b) \cdot c$




$a \cdot b \cdot c$

# Lab 4 : Examples

| | | | | | |
|---|---|---|---|---|---|
| n | 100 | 200 | 400 | 800 | 1600 |
| f(n) | 113979 | 527482 | 2391840 | 10685800 | 47198500 |

**Guess $\Theta(n^3)$**

| | | | | | |
|---|---|---|---|---|---|
| n^3 | 1000000 | 8000000 | 64000000 | 512000000 | 4096000000 |
| f(n)/n^3 | 0.113979 | 0.0659352 | 0.0373724 | 0.0208708 | 0.0115231 |

Conclusion: $\Theta(n^3)$ Too high.

**Guess $\Theta(n^2)$**

| | | | | | |
|---|---|---|---|---|---|
| n^2 | 10000 | 40000 | 160000 | 640000 | 2560000 |
| f(n)/n^2 | 11.3979 | 13.187 | 14.949 | 16.6966 | 18.4369 |

Conclusion: $\Theta(n^2)$ Too low.

**Guess $\Theta(n^2 \log n)$**

| | | | | | |
|---|---|---|---|---|---|
| n^2log(n) | 46051.7 | 211933 | 958634 | 4278150 | 18887100 |
| f(n)/n^2log(n) | 2.47503 | 2.48891 | 2.49505 | 2.49777 | 2.49899 |

Conclusion: $f(n) = \Theta(n^2 \log n)$

$\text{const} = 2.498$