CMAS 10 : 10-27-09

Binary Search

1.) $L \leftarrow 1$
2.) $R \leftarrow n$
3.) found $\leftarrow$ false
4.) while $L \leq R$ and not found
5.) $\quad m \leftarrow \left\lfloor \frac{L+R}{2} \right\rfloor$
6.) $\quad$ if target $= a_m$
7.) $\quad\quad$ found $\leftarrow$ true
8.) $\quad$ else if target $< a_m$
9.) $\quad\quad R \leftarrow m-1$
10.) $\quad$ else
11.) $\quad\quad L \leftarrow m+1$
12.) if not found
13.) $\quad m \leftarrow 0$
14.) Print m
15.) stop

Ex. $n = 10$, target $= 8$

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
2	3	4	7	10	11	15	20	25	28

L	R	m	found
~~1~~	~~10~~	~~5~~	F
~~3~~	4	~~2~~	
~~4~~		~~3~~	
5		~~4~~	
		$\boxed{0}$	

Value Printed

Trace of Quiz 2 #1:

i	j	Print
1	1	h
2	2	
3	1	h
4	2	h
5	3	
	1	h
	2	h
	3	h
	4	
	1	h
	2	h
	3	h
	4	h
	5	

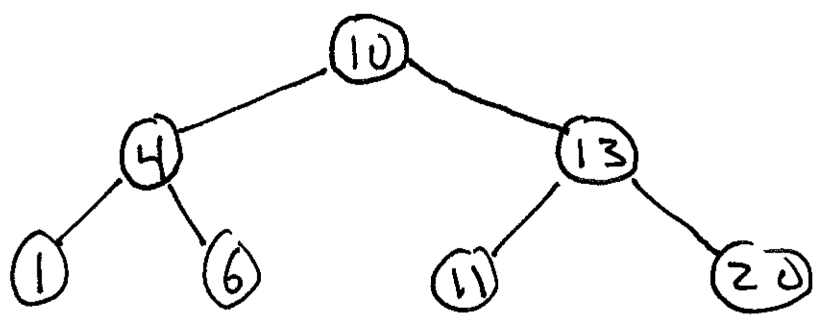
Runtime of Binary Search.

Basic op: Comparison of target to a list element.

Binary Search tree:

Ex 1 4 6 10 11 13 20

depth = 3

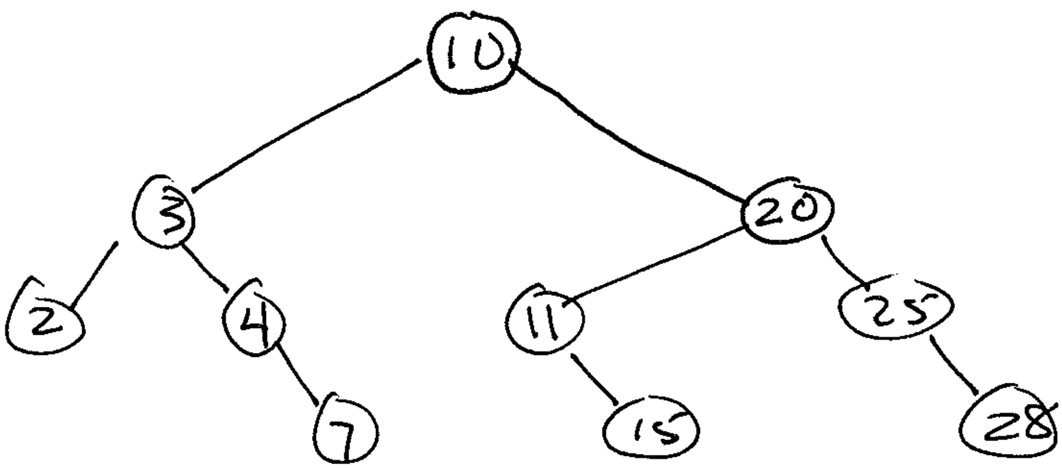


comp
1
2
3

- Depth in tree at which target is located is # comp needed to find that target.
- If target is not in list, # comp is no more than depth of tree.

Ex.

Ind:	1	2	3	4	5	6	7	8	9	10	
	2	3	4	7	10	11	15	20	25	28	# comp



1

2

3

4

- for this list, worst case # comp is $\boxed{4}$

- Assume target is in list, and equally likely to be any position: then avg. case # comp is:

$$\frac{1\cdot 1 + 2\cdot 2 + 4\cdot 3 + 3\cdot 4}{10} = \frac{29}{10} = \boxed{2.9}$$

Let $W(n)$ = worst case # comp
performed by B.S. on
any list of length n.
(note: $W(10)=4$ by last ex.)

Since list contents don't matter in this analysis, we assume our list of length n is:

1 2 3 . . . . n

Observe:

$n=3$: 1 2 3 # comp

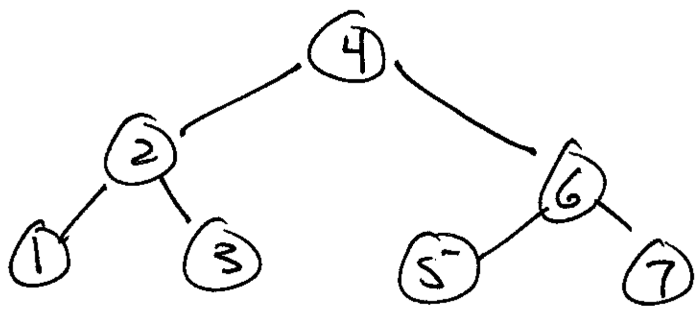
$3 = 2^{2}-1$

1
2

$W(3) = 2$.

$n = 7$:

$7 = 2^{\boxed{3}} - 1$ 1 2 3 4 5 6 7

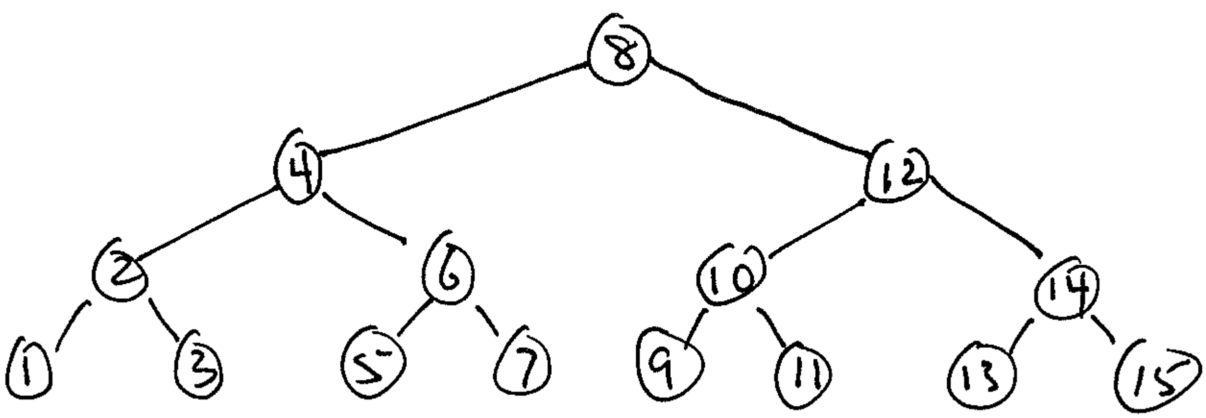


comp
1
2
3

$$W(7) = \boxed{3}$$

$n = 15$: $15 = 2^{\boxed{4}} - 1$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15



comp
1
2
3
4

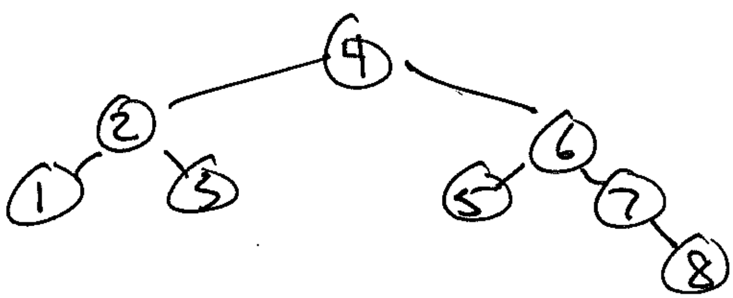
$$W(15) = \boxed{4}$$

<u>GOAL</u>: write a formula for $W(n)$

<u>EXERCISE</u>: Draw BST's for some of 'missing' values for n.

$n = 8$:

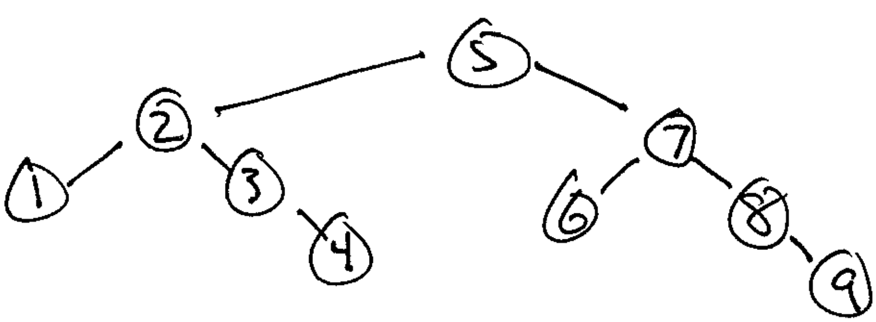
1 2 3 4 5 6 7 8



$W(8) = 4$

$n = 9$:

1 2 3 4 5 6 7 8 9



$W(9) = 4$

Theorem

If $2^{K-1} - 1 < n \le 2^{K} - 1$, then

$W(n) = K$.

So for instance

$W(100) = \boxed{7}$ since

$63 < 100 \le 127 = 2^{\boxed{7}} - 1$

Defn

Let $b > 1$, $x > 0$. Then

$\log_b(x)$ = the power you must raise b to, to get x.

Ex. $\log_3 9 = 2$ since $3^2 = 9$

$\log_5 125 = 3$ since $5^3 = 125$

$\log_{10}(10000) = 4$ since $10^4 = 10000$

$\log_2(32) = 5$ since $2^5 = 32$

In general

$$\boxed{y = \log_b x \quad \text{iff} \quad x = b^y}$$

Special bases:

notation

<u>Common log</u>: $b=10$: $\log_{10} = \log$

<u>NATURAL log</u>: $b=e=2.71828\ldots$ $\log_e = \ln$

<u>Binary log</u>: $b=2$ $\log_2 = \lg$