CMPS 10 10-22-09

Summary of Sorting Run times:

 # list comparisons:

	Best	avg	* worst
Selection:	$\frac{n(n-1)}{2}$	$\frac{n(n-1)}{2}$	$\frac{n(n-1)}{2}$
Bubble:	$\frac{n(n-1)}{2}$	$\frac{n(n-1)}{2}$	$\frac{n(n-1)}{2}$
Insertion:	$n-1$	??	$\frac{n(n-1)}{2}$

will ~~ATTACK~~ this experimentally in lab4

NOTE: worst case was always

$$\frac{n(n-1)}{2} = \frac{1}{2}n^2 - \frac{1}{2}n = \Theta(n^2)$$

If we counted peripheral ops.
we would get

$$\text{worst case \# comp} = \underline{a}\,n^2 + \underline{b}\,n + \underline{c}$$

constants that are
machine dependent

wish to disregard these coeffs
and concentrate on "n^2"

ASYMPTOTIC GROWTH RATE of A function (Informal Defn):

let $f(n)$, $g(n)$ be functions.

write $\underbrace{f(n) = \Theta(g(n))}$

read 'f(n) is of order g(n)'

<u>if</u> $\boxed{f(n) = \text{const} \cdot g(n) + (\text{lower order terms})}$

i.e. $\frac{L.O.T(n)}{g(n)} \to 0$ as $n \to \infty$

Ex. $2n^2 + 3n + 5 = \Theta(n^2)$

$$\frac{3n+5}{n^2} = \frac{3}{n} + \frac{5}{n^2} \longrightarrow 0$$

$$\begin{array}{cc} \frac{3}{n} & \frac{5}{n^2} \\ \downarrow & \downarrow \\ 0 & 0 \end{array}$$

Ex. $100n^2 + 12n + \sqrt{n} - 3 = \Theta(n^2)$

Ex. $5n^3 - n + 100 = \Theta(n^3)$

Ex. $3\sqrt{n} + \sqrt[3]{n} + \sqrt[4]{n}$

$$= 3 \cdot n^{1/2} + n^{1/3} + n^{1/4} = \Theta(n^{1/2})$$

Ex. $5n\sqrt{n} + 100n^2 + \sqrt{n}$

$$5 \cdot n^{3/2} + 100n^2 + n^{1/2} = \Theta(n^2$$

Ex. $20n^{7/3} + n^{5/2} + \sin(n) = \Theta(n^{5/2})$

highest order term

$\frac{5}{2} > \frac{7}{3}$

$15 > 14$

what about logarithms?

$\log_b(n)$

will answer later.

Summarize again.

	best	avg	worst
selection	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$
Bubble	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$
Insertion	$\Theta(n)$	??	$\Theta(n^2)$

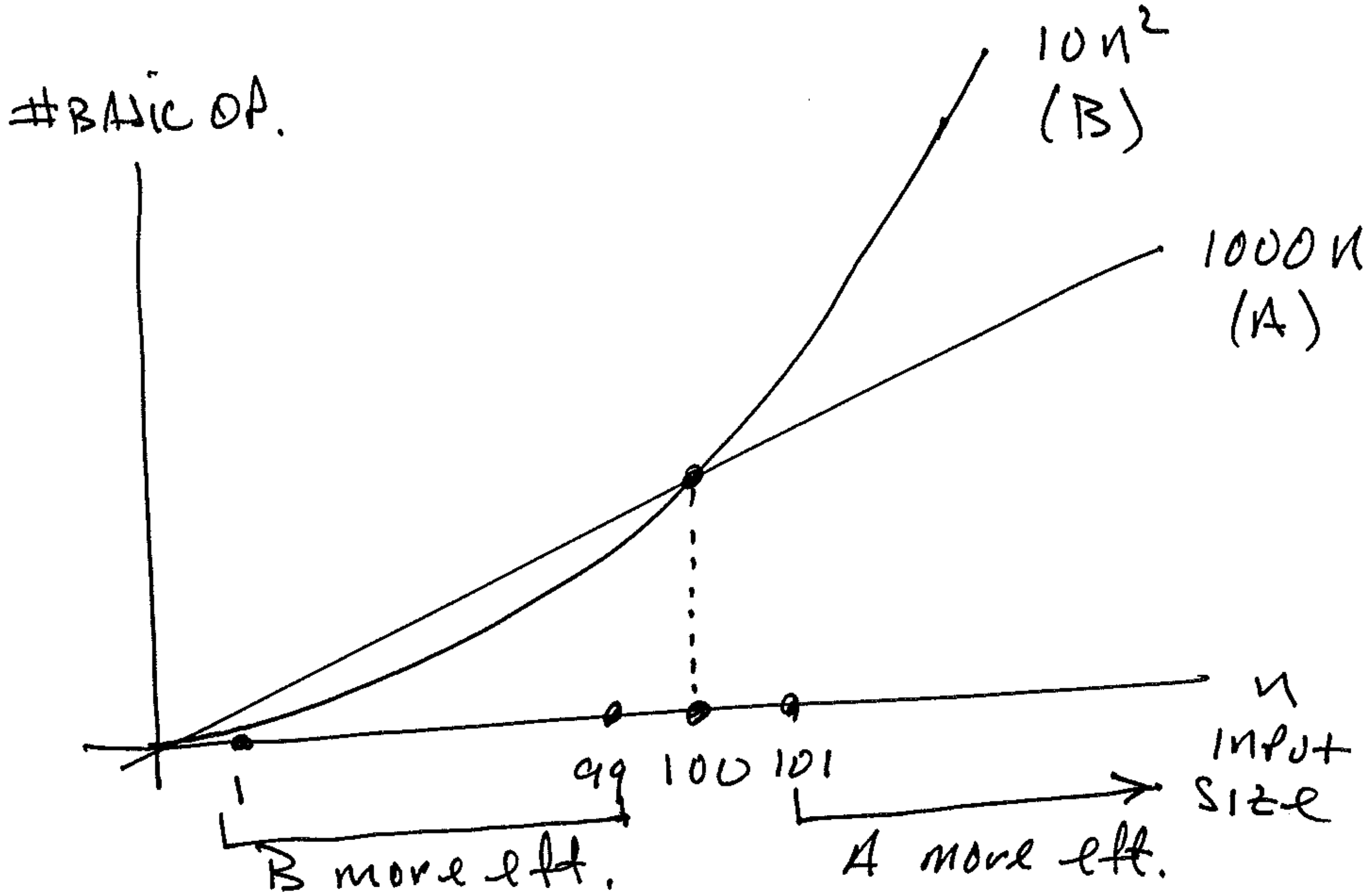
experiment

Ex. Say have 4 Algorithms: A, B, C, D, which do same thing. Suppose worst case # Basic ops are:

$$
\begin{array}{lll}
A: & 1000n & \} \ \Theta(n) \\
B: & 10n^2 & \\
C: & n^2 & \Theta(n^2) \\
D: & n^2 + 100n + 1000 &
\end{array}
$$

- B & C can be equalized by running B on a faster computer
- lower order terms in D are negligible for large n.

- compare A & B



A & B are equally efficient when $n = 100$.

why? $10n^2 = 1000n$ solve for n

$n = 100$ ✓

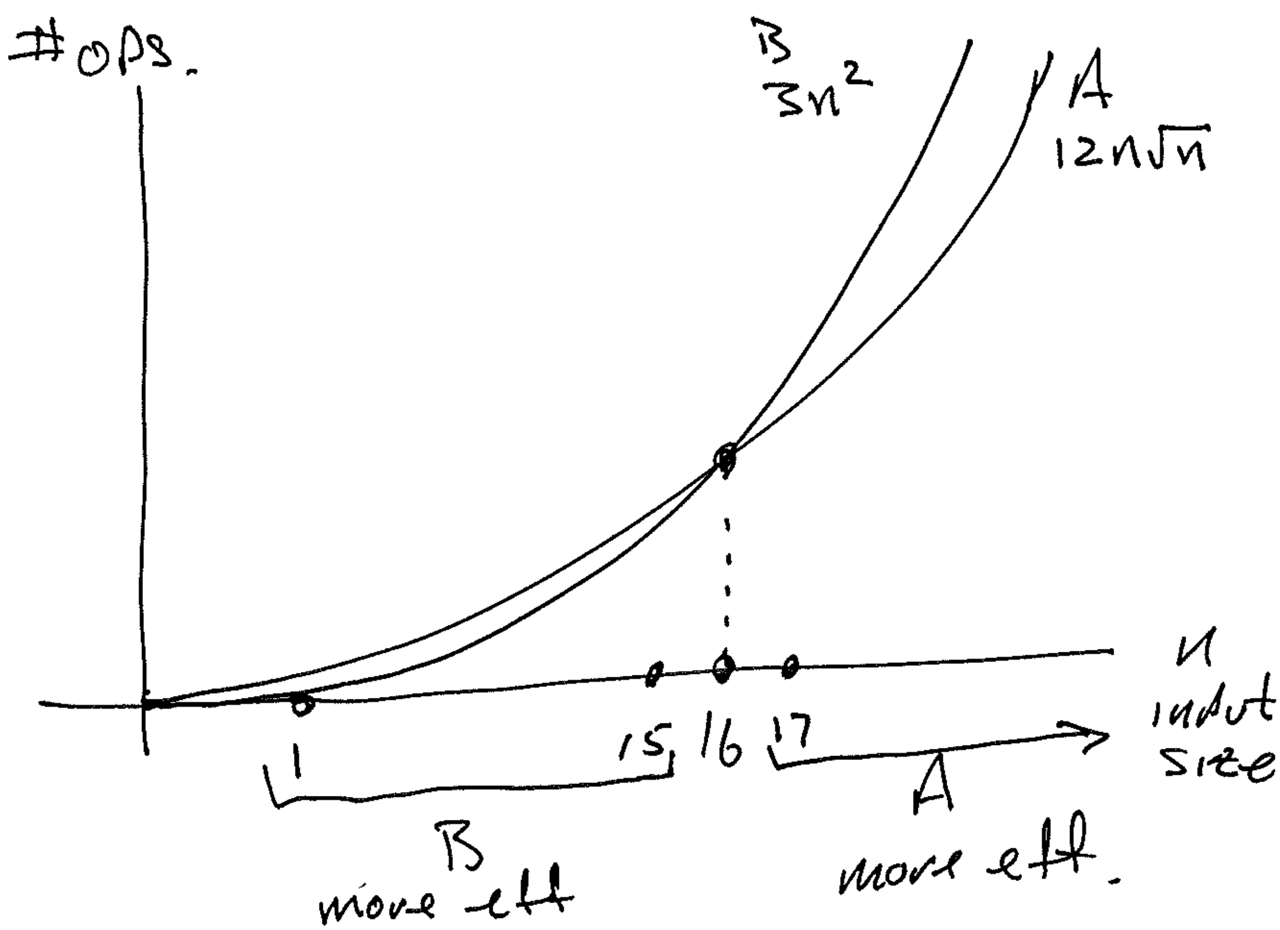
Ex.

A: $12n\sqrt{n}$ BASIC OPS. $\Theta(n^{3/2})$

B: $3n^2$ BASIC OPS. $\Theta(n^2)$

For which n are A & B equally efficient?

$$12n\sqrt{n} = 3n^2 \Rightarrow 4 = \sqrt{n} \Rightarrow 16 = n$$



Binary Search

Ex. $n = 10$, target $= 11$

1	2	3	4	5	6	7	8	9	10
2	3	4	7	10	11	15	20	25	28

$$\underset{L}{\big|}\; 2 \quad 3 \quad 4 \quad 7 \quad 10 \quad 11 \quad 15 \quad 20 \quad 25 \quad 28 \;\underset{R}{\big|}$$

$$2 \quad 3 \quad 4 \quad 7 \quad 10 \;\underset{L}{\big|}\; 11 \quad 15 \quad 20 \quad 25 \quad 28 \;\underset{R}{\big|}$$

$$2 \quad 3 \quad 4 \quad 7 \quad 10 \;\underset{L}{\big|}\; 11 \quad 15 \;\underset{R}{\big|}\; 20 \quad 25 \quad 28$$

$$\left(\begin{array}{l}\#\text{ comparisons of target to a} \\ \text{list element}\end{array}\right) = 3$$

Ex. $n = 10$, target $= 3$

| 2 3 4 7 10 11 15 20 25 28 |

L R

| 2 3 4 7 | 10 11 15 20 25 28

L R

comp = 2

Ex. $n = 10$ target $= 7$

| 2 3 4 7 10 11 15 20 25 28 |

L R

| 2 3 4 7 |

L R

| 4 7 |

L R

| 7 |

L R

#comp = 4

Ex. $n=10$, target $=8$

| 2 3 4 7 10 11 15 20 25 28 |

L R

| 2 3 4 7 |

L R

| 4 7 |

L R

| 7 |

L R

| |

R L

Quit!

comp = 4

Input: $n \geq 1$, $\underbrace{a_1, \ldots, a_n}_{\text{sorted increasing}}$, target

Output: 'the' index m s.t. $a_m =$ target, or 0 if no such m exists.

Defn: $\lfloor x \rfloor$ = floor of x
= greatest integer which is $\leq x$
= integer part of x.

i.e. $\lfloor 4.5 \rfloor = 4$

$\lfloor 4 \rfloor = 4$

$\lfloor -5.9 \rfloor = -6$

Binary Search

1.) $L \leftarrow 1$

2.) $R \leftarrow n$

3.) found $\leftarrow$ false

4.) while $L \leq R$ and not found

5.) $\quad m \leftarrow \left\lfloor \frac{L+R}{2} \right\rfloor$

$\vdots$