CMPS 10 10-20-09

Bubble Sort:

Ex. $n=6$

	R
7 9 5 0 4 3 |	6
7 5 9 0 4 3	
7 5 0 9 4 3	
7 5 0 4 9 3	
7 5 0 4 3 | 9	5
5 7 0 4 3	
5 0 7 4 3	
5 0 4 7 3	
5 0 4 3 | 7 9	4
0 5 4 3	
0 4 5 3	
0 4 3 | 5 7 9	3
0 3 | 4 5 7 9	2
0 | 3 4 5 7 9	1

Input: $n \geq 1$, $a_1, \ldots, a_n$

OUTPUT: modified list in inc. order

```
1.) R ← n
2.) while R ≥ 2
3.)     j ← 2
4.)     while j ≤ R
5.)         if a_j < a_{j-1}          ← Basic Op.
6.)             a_j ↔ a_{j-1}  (swap)
7.)         j ← j+1
8.)     R ← R-1
9.) stop
```

Refinement of (6):

```
(6.1) temp ← a_j
(6.2) a_j ← a_{j-1}
(6.3) a_{j-1} ← temp
```

Run Time

Count # of comparisons of list elements in best, worst, & avg. cases.

outer	inner	# comp
$k=n$:	$2 \le j \le n$	$n-1$
$k=n-1$:	$2 \le j \le n-1$	$n-2$
$k=n-2$:	$2 \le j \le n-2$	$n-3$
⋮		
$k=3$:	$2 \le j \le 3$	2
$k=2$:	$2 \le j \le 2$	1

total # of comp $= 1+2+\cdots+n-1$

$$= \frac{n(n-1)}{2} = \frac{1}{2}n^2 - \frac{1}{2}n$$

Insertion Sort

index of leftmost elt. in unsorted seq.
"
L

Ex. $n = 6$

						L
2	4	6	1	5	3	2
2	4	6	1	5	3	3
2	4	6	1	5	3	4
1	2	4	6	5	3	5
1	2	4	5	6	3	6
1	2	3	4	5	6	7

Insertion Sort

1.) $L \leftarrow 2$

2.) while $L \leq n$

3.) $\quad j \leftarrow L$

4.) $\quad$ while $j \geq 2$ and $a_j < a_{j-1}$ ← BASIC OP.

5.) $\quad\quad a_j \leftrightarrow a_{j-1}$ (swap)

6.) $\quad\quad j \leftarrow j-1$

7.) $\quad L \leftarrow L+1$

8.) stop

Runtime:

Best case: assume $a_1 \cdots a_n$ is already sorted.

	#comp
$a_1 \mid a_2 \; a_3 \cdots a_{n-1} \; a_n$	1
$a_1 \; a_2 \mid a_3 \cdots a_{n-1} \; a_n$	1
$a_1 \; a_2 \; a_3 \mid a_4 \cdots a_{n-1} \; a_n$	1
$\vdots$	
$a_1 \; a_2 \; a_3 \; a_4 \cdots a_{n-1} \mid a_n$	1

Best case # comp = $n-1$

<u>Worst Case</u>: assume $a_1 \cdots a_n$ is sorted in descending (rather than asc.) order.

	# comp
$a_1 \mid a_2\ a_3 \cdots a_{n-1}\ a_n$	1
$a_1\ a_2 \mid a_3 \cdots a_{n-1}\ a_n$	2
$a_1\ a_2\ a_3 \mid a_4 \cdots a_{n-1}\ a_n$	3
$\vdots$	
$a_1\ a_2\ a_3\ a_4 \cdots a_{n-1} \mid a_n$	$n-1$

Worst case # comps =

$$= 1 + 2 + 3 + \cdots + (n-1) = \frac{n(n-1)}{2}$$

$$= \frac{1}{2}n^2 - \frac{1}{2}n$$

Average case! ??

will Determine this experimentally in future lab assignment.