CMPS 10 10-31-08

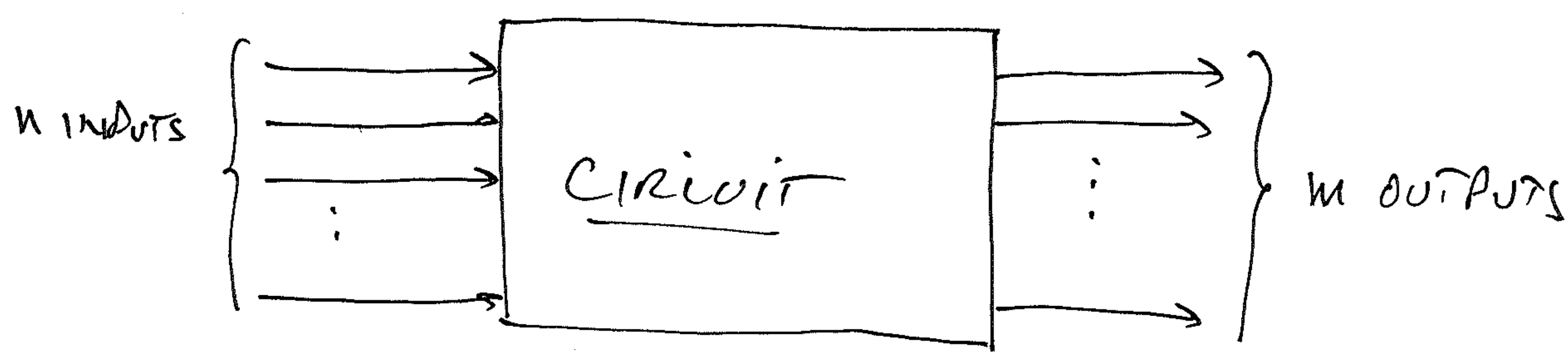
L1

logical EQUIVALENCE

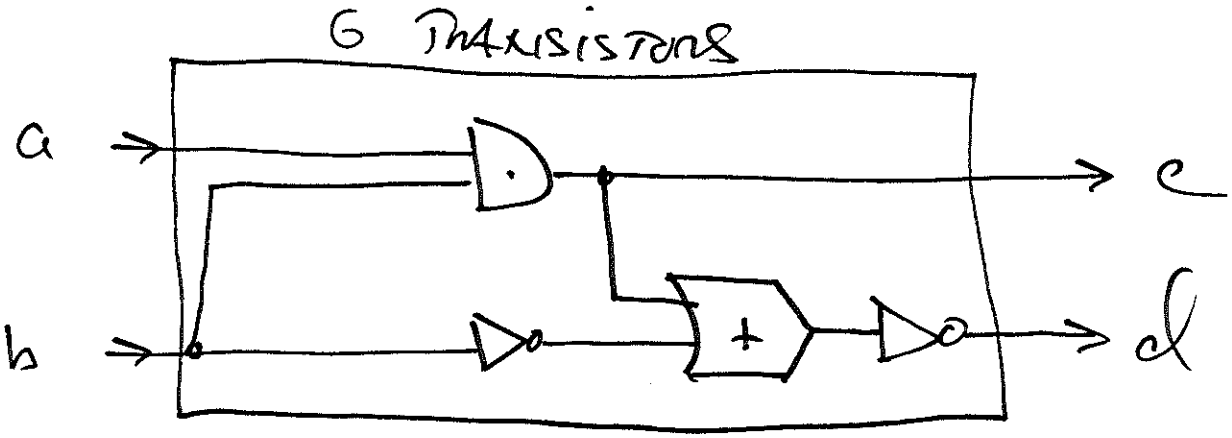
NOTICE : $p \oplus q \equiv p \cdot \bar{q} + \bar{p} \cdot q$

p	q	$\bar{p}$	$\bar{q}$	$p \cdot \bar{q}$	$\bar{p} \cdot q$	$p \cdot \bar{q} + \bar{p} \cdot q$	$p \oplus q$
0	0	1	1	0	0	0	0
0	1	1	0	0	1	1	1
1	0	0	1	1	0	1	1
1	1	0	0	0	0	0	0

DEFN: A ***COMBINATIONAL CIRCUIT*** IS A COLLECTION OF LOGIC GATES THAT TRANSFORMS A SET OF BINARY INPUTS TO A SET OF BINARY OUTPUTS. ***REQUIRE***: OUTPUTS DEPEND ONLY ON CURRENT OF INPUTS.



Ex.



$d \equiv \overline{(a \cdot b) + \bar{b}}$

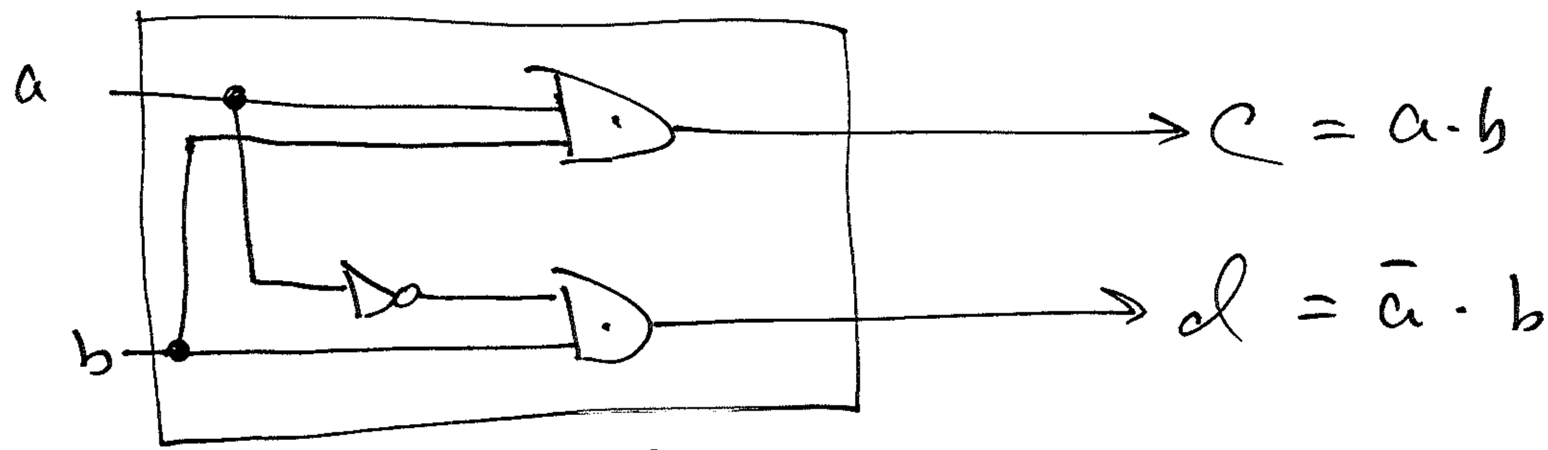
a	b	c	d
0	0	0	0
0	1	0	1
1	0	0	0
1	1	1	0

NOTE:

$c \equiv a \cdot b$

$d \equiv \bar{a} \cdot b$

THIS SUGGESTS AN ALTERNATIVE DESIGN:



5 TRANSISTORS

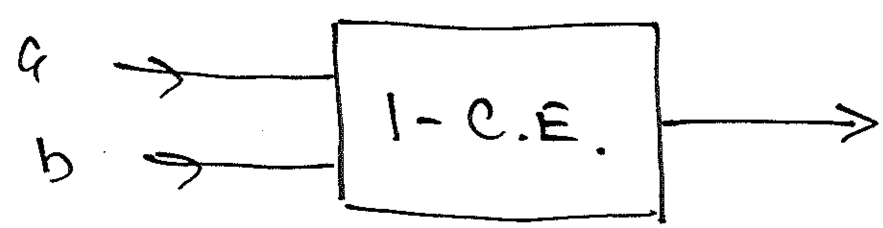
LOGICAL EQUIVALENCE: $\overline{(a \cdot b) + \bar{b}} \equiv \bar{a} \cdot b$

a	b	$\bar{a}$	$\bar{b}$	$a \cdot b$	$\bar{a} \cdot b$	$(a \cdot b) + \bar{b}$	$\overline{(a \cdot b) + \bar{b}}$
0	0	1	1	0	0	1	0
0	1	1	0	0	1	0	1
1	0	0	1	0	0	1	0
1	1	0	0	1	0	1	0

CORRESPONDENCE:

$$\begin{tikzcd}
\text{CIRCUITS} \arrow[rr, leftrightarrow, "\text{1 TO 1}"] \arrow[dr, "\text{MANY TO 1}"'] & & \text{LOGICAL EXPRESSIONS} \arrow[dl, "\text{MANY TO 1}"] \\
& \text{TRUTH TABLES} &
\end{tikzcd}$$

EX. 1-BIT COMPARE FOR EQUALITY CIRCUIT.



$$c = \begin{cases} 1 & \text{if } a = b \\ 0 & \text{if } a \neq b \end{cases}$$

TRUTH TABLE:

a	b	c	$a \cdot b$	$\bar{a}$	$\bar{b}$	$\bar{a} \cdot \bar{b}$	$\bar{a} \cdot \bar{b} + a \cdot b$
0	0	1	0	1	1	1	1
0	1	0	0	1	0	0	0
1	0	0	0	0	1	0	0
1	1	1	1	0	0	0	1

NOTE: $c \equiv \bar{a} \cdot \bar{b} + a \cdot b$ ✓

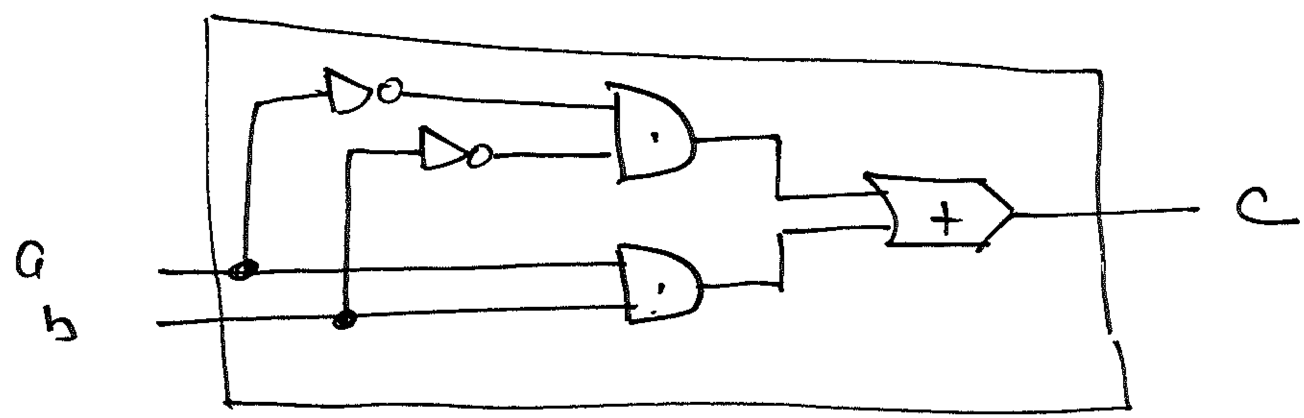
CONVENTION:

- $\cdot$ HAS HIGHER PRECEDENCE THAN $+$ SO

$a \cdot b + c$

means $(a \cdot b) + c$

NOT $a \cdot (b + c)$



TRANSISTORS = 8

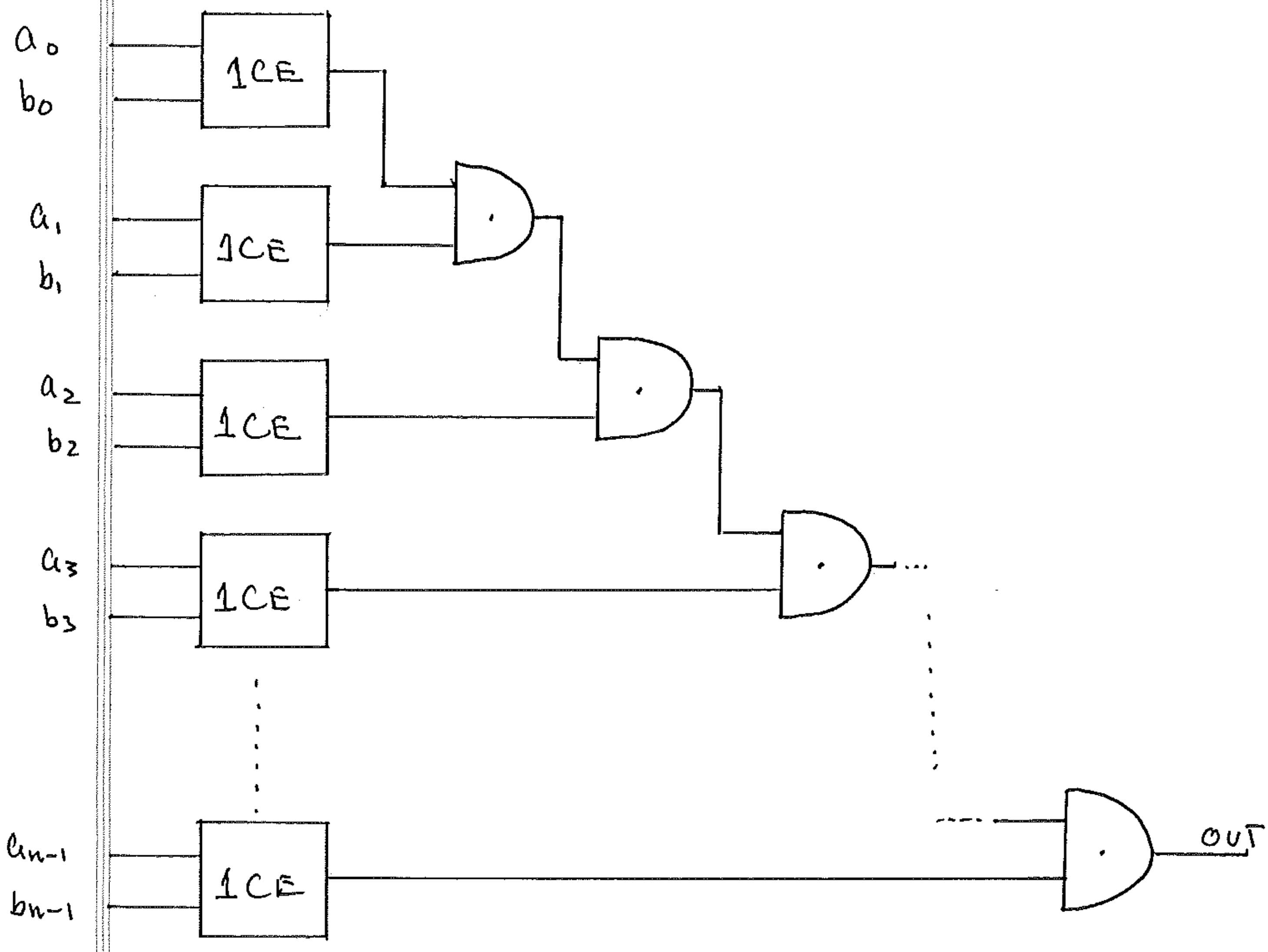
EX. n-BIT COMPARER FOR EQUALITY CIRCUIT.

INPUT:

$$\left.\begin{array}{l} a = a_{n-1} \cdots a_1 a_0 \\ b = b_{n-1} \cdots b_1 b_0 \end{array}\right\} \text{TWO n-BIT BINARY \#S}$$

OUTPUT: SINGLE BIT $c = \begin{cases} 1 & \text{if } a = b \\ 0 & \text{if } a \neq b \end{cases}$

INPUTS: TWO n BIT BINARY NUMBERS
$[a_{n-1} \cdots a_0]_2$, $[b_{n-1} \cdots b_0]_2$
OUTPUT: 1 IF EACH $a_i = b_i$ $(0 \le i \le n-1)$,
0 OTHERWISE



HOW MANY ROWS AND COLUMNS WOULD THE TRUTH TABLE FOR THIS CIRCUIT HAVE ?

ANSWER: 2^{2n} ROWS
$2n+1$ COLUMNS

EX. GOAL: n-BIT full ADDER

$n=8$:

$$
\begin{array}{r}
1\,0\,1\,1\,0\,1\,1\,0\,0 \\
1\,0\,0\,1\,0\,1\,1\,0 \\
1\,0\,1\,1\,0\,0\,1\,1 \\
\hline
1\,0\,1\,0\,0\,1\,0\,0\,1
\end{array}
$$