BINARY SEARCH

AVERAGE CASE RUNTIME: ASSUME target in list; EQUALLY LIKELY TO BE IN ANY POSITION.

$$A(n) = \frac{\left(\sum_{i=1}^{k} i \cdot 2^{i-1}\right)}{n} = \lg(n+1) - 1 + \frac{\lg(n+1)}{n}$$

EASY HARD

FOR $\boxed{n = 2^k - 1}$

STANDS FOR $1 \cdot 2^{1-1} + 2 \cdot 2^{2-1} + 3 \cdot 2^{3-1} + \cdots + k \cdot 2^{k-1}$

$\therefore \quad A(n) = \Theta(\lg n)$

CHAP 4: LOGIC & CIRCUITS

THE <u>BASE b</u> POSITIONAL NUMBERING SYSTEM.

<u>EX</u> WHAT DOES 12526 MEAN ?

$$(12526)_{10} = 1\cdot 10^4 + 2\cdot 10^3 + 5\cdot 10^2 + 2\cdot 10^1 + 6\cdot 10^0$$

GIVEN AN INTEGER $b>1$, ASSIGN b SYMBOLS (CALLED DIGITS) TO THE #S

$$0, 1, 2, \ldots, b-1$$

A STRING OF DIGITS

$$a_{n-1}\, a_{n-2} \cdots a_3\, a_2\, a_1\, a_0$$

STANDS FOR:

$$\underbrace{(a_{n-1} \cdots a_1 a_0)_b}_{n \text{ DIGITS}} = a_{n-1} \cdot b^{n-1} + a_{n-2} \cdot b^{n-2} + \cdots + a_2 \cdot b^2 + a_1 \cdot b^1 + a_0 \cdot b^0$$

MORE GENERALLY

$$(a_{n-1} \cdots a_1 a_0 \,.\, a_{-1} a_{-2} \cdots a_{-k})_b = a_{n-1} \cdot b^{n-1} + \cdots + a_0 \cdot b^0 + a_{-1} \cdot b^{-1} + a_{-2} \cdot b^{-2} + \cdots + a_{-k} \cdot b^{-k}$$

"base b Point"

IN C.S WE ARE CONCERNED WITH

$b=2$: BINARY DIGITS $\{0, 1\}$

$b=8$: OCTAL DIGITS $\{0, 1, 2, 3, 4, 5, 6, 7\}$

$b=16$: HEXIDECIMAL $\{0, 1, 2, \ldots, 9, \underset{\substack{\shortparallel\\10}}{A}, \underset{\substack{\shortparallel\\11}}{B}, \underset{\substack{\shortparallel\\12}}{C}, \underset{\substack{\shortparallel\\13}}{D}, \underset{\substack{\shortparallel\\14}}{E}, \underset{\substack{\shortparallel\\15}}{F}\}$

$b=10$: DECIMAL $\{0, 1, \ldots, 9\}$

<u>EXERCISE</u>: COUNT IN BASE 2:

0, 1, 10, 11, 100, 101, 110, 111, 1000, 1001, ...

<u>Bin</u>	<u>Dec</u>
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	10
1011	11
1100	12
1101	13
1110	14
1111	15

Ex.

$$(1011010111001)_2 = (\,?\,)_{10}$$

$$= 1\cdot 2^{12} + \cancel{0\cdot 2^{11}} + 1\cdot 2^{10} + 1\cdot 2^{9} + \cancel{0\cdot 2^{8}} + 1\cdot 2^{7} + \cancel{0\cdot 2^{6}} + 1\cdot 2^{5} + 1\cdot 2^{4} + 1\cdot 2^{3} + \cancel{0\cdot 2^{2}} + \cancel{0\cdot 2^{1}} + 1\cdot 2^{0}$$

$$= 2^{12} + 2^{10} + 2^{9} + 2^{7} + 2^{5} + 2^{4} + 2^{3} + 2^{0}$$

$$= 4096 + 1024 + 512 + 128 + 32 + 16 + 8 + 1$$

$$= (5817)_{10}$$

Ex. $(237)_8 = (\,?\,)_{10}$

$$= 2\cdot 8^{2} + 3\cdot 8^{1} + 7\cdot 8^{0} = 2\cdot 64 + 3\cdot 8 + 7 = 128 + 24 + 7$$

$$= (159)_{10}$$

Ex. $(A17D)_{16} = 10\cdot 16^3 + 1\cdot 16^2 + 7\cdot 16^1 + 13\cdot 16^0$

$= \dots = (41341)_{10}$

BASE 10 → BASE 2:

Ex. $(357)_{10} = (\,?\,)_2$

k	0	1	2	3	4	5	6	7	8	9
2^k	1	2	4	8	16	32	64	128	256	512

$357 = 256 + 101 = 256 + 64 + 37 = 256 + 64 + 32 + 5$

$= 256 + 64 + 32 + 4 + 1 = 2^8 + 2^6 + 2^5 + 2^2 + 2^0$

$= \underline{1}\cdot 2^8 + \underline{0}\cdot 2^7 + \underline{1}\cdot 2^6 + \underline{1}\cdot 2^5 + \underline{0}\cdot 2^4 + \underline{0}\cdot 2^3 + \underline{1}\cdot 2^2 + \underline{0}\cdot 2^1 + \underline{1}\cdot 2^0$

$= (101100101)_2$