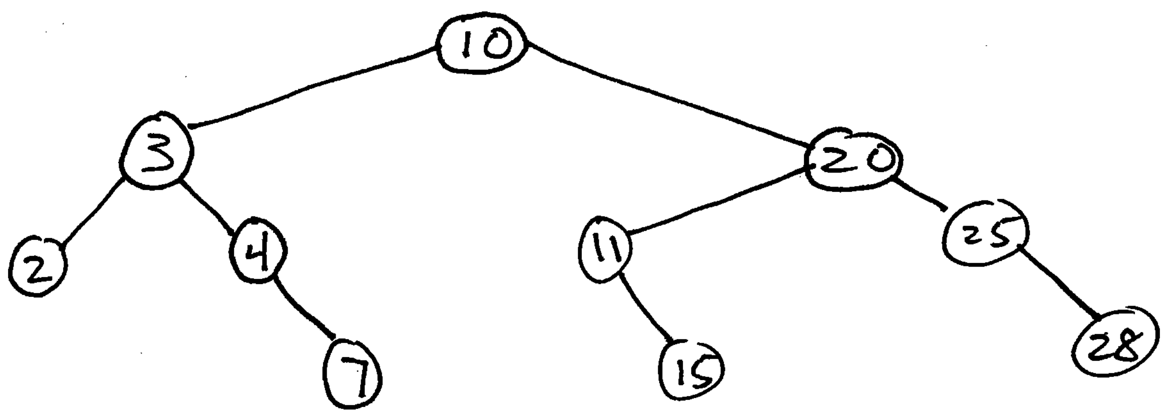
CMPS 10 10-20-08

EX. $n=10$

IND.	1	2	3	4	5	6	7	8	9	10
	2	3	4	7	10	11	15	20	25	28



\# COMP
1
2
3
$\boxed{4}$

WORST CASE # COMP $= 4$

AVG. CASE # COMP $= \dfrac{1\cdot 1 + 2\cdot 2 + 4\cdot 3 + 3\cdot 4}{10} = \boxed{2.9}$

ASSUME TARGET IS IN LIST.

LET $W(n)$ = WORST CASE # OF COMPARISONS PERFORMED BY BIN. SEARCH ON LISTS OF LENGTH n.

KNOW: $W(10) = 4$, $W(7) = 3$

TO FIND $W(n)$: SEARCH "STANDARD" LIST

$$\boxed{1 \quad 2 \quad 3 \quad \cdots\cdots \quad n-1 \quad n}$$

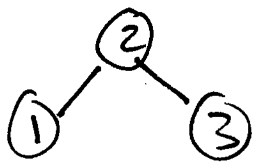
WILL FIND A GENERAL FORMULA FOR $W(n)$

$\underline{n=1}$: 1 $w(1) = 1$

$1 = 2^{1} - 1$

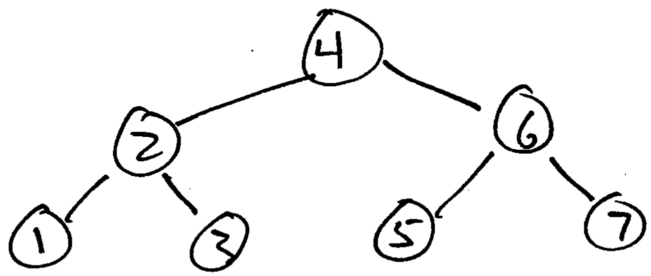
$\underline{n=3}$: 1 2 3 $w(3) = 2$

$3 = 2^{2} - 1$



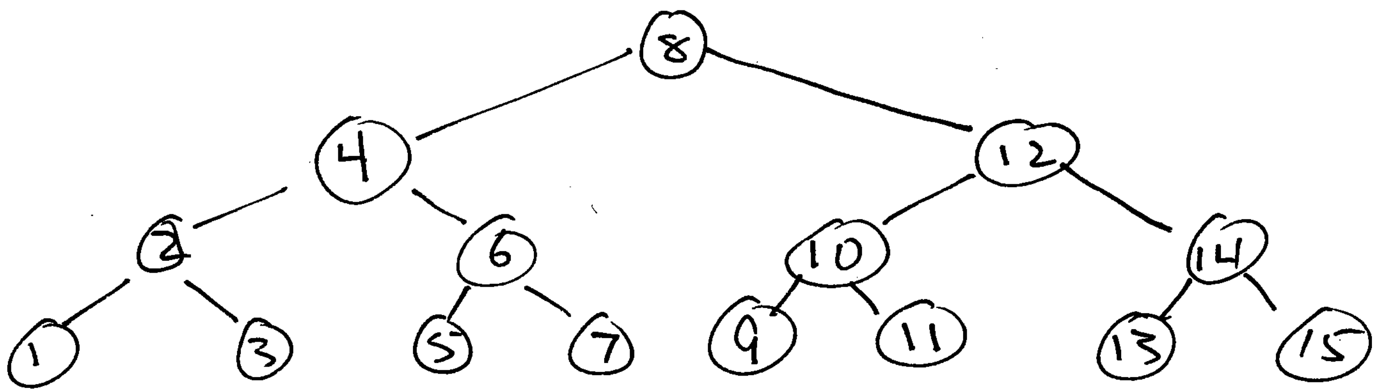
$\underline{n=7}$: 1 2 3 4 5 6 7 $w(7) = 3$

$7 = 2^{3} - 1$



$\underline{n = 15}$: $15 = 2^{4} - 1$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15



$w(15) = 4$

$\underline{n = 31}$: EXERCISE. $w(31) = 5$

$31 = 2^{5} - 1$

MAGIC NUMBERS : $\underbrace{n = 2^{k} - 1}$

⟸ CORRESPONDING BINARY TREE is COMPLETE.

THM IF $n = 2^K - 1$, THEN $W(n) = K$

THM IF $2^{K-1} - 1 < n \le 2^K - 1$, THEN $W(n) = K$.

base of the log function

DEFN: LET $b > 1$, $x > 0$. $\log_b(x)$ IS THE POWER YOU MUST RAISE b TO, TO GET x. i.e.

$$y = \log_b x \quad \underline{\underline{\text{IFF}}} \quad x = b^y$$

i.e. $\log_b(b^y) = y$ AND $b^{\log_b x} = x$

Ex.

$\log_3 9 = 2$ SINCE $3^2 = 9$

$\log_5 125 = 3$ " $5^3 = 125$

$\log_{10}(10000) = 4$ " $10^4 = 10000$

$\log_2(32) = 5$ " $2^5 = 32$

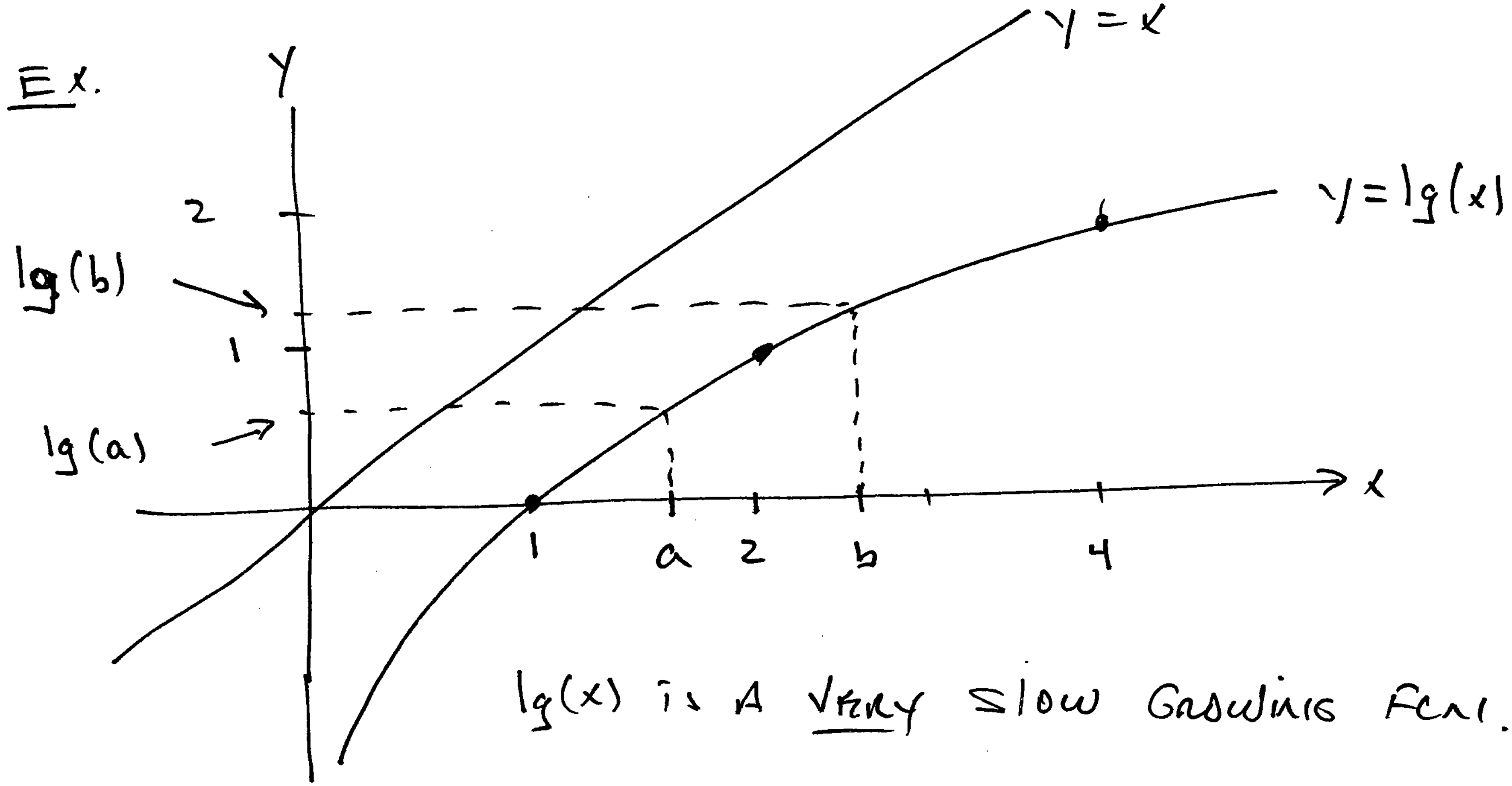
$\log_{10}(x) = \log(x)$: <u>COMMON LOG</u>, <u>NAPERIAN LOG</u>

$\log_e(x) = \ln(x)$: <u>NATURAL LOG</u>.

$e = 2.71828\ldots$

$\log_2(x) = \lg(x)$: <u>BINARY LOG</u>.

Ex.



lg(x) is a <u>very</u> slow growing fcn.

$a < b$ **iff** $\lg(a) < \lg(b)$

RECALL: $2^{K-1} - 1 < n \le 2^{K} - 1$ **IFF** $W(n) = K$

$\therefore \quad 2^{K-1} \le n < 2^{K}$

$\therefore \quad \lg(2^{K-1}) \le \lg(n) < \lg(2^{K})$

$\therefore \quad K-1 \le \lg(n) < K$

$\therefore \quad K-1 = \lfloor \lg(n) \rfloor$

$\therefore \quad K = \lfloor \lg(n) \rfloor + 1$

THM: $\boxed{W(n) = \lfloor \lg(n) \rfloor + 1}$

COR: $W(n) = \Theta(\lg(n))$