CMPS 10 10-15-08

INSERTION SORT

EX

						L
2 |	4	6	1	5	3	2
2	4 |	6	1	5	3	3
2	4	6 |	1	5	3	4
2	4	1 |	6	5	3	
2	1	4 |	6	5	3	
1	2	4	6 |	5	3	5
1	2	4	5	6 |	3	6
1	2	4	5	3	6	
1	2	4	3	5	6	
1	2	3	4	5	6 |	T

INSERTION SORT

1.) $L \leftarrow 2$

2.) while $L \le n$

3.) $\quad j \leftarrow L$

4.) $\quad$ while $j \ge 2$ and $a_j < a_{j-1}$ ← BASIC OPERATION

5.) $\quad\quad$ swap $a_j \leftrightarrow a_{j-1}$

6.) $\quad\quad j \leftarrow j-1$

7.) $\quad L \leftarrow L+1$

8.) stop

Run Time of Insertion Sort

	# Comp Best	# Comp Worst
$a_1 \mid a_2 \; a_3 \; a_4 \cdots a_{n-1} \; a_n$	1	1
$a_1 \; a_2 \mid a_3 \; a_4 \cdots a_{n-1} \; a_n$	1	2
$a_1 \; a_2 \; a_3 \mid a_4 \cdots a_{n-1} \; a_n$	1	3
$\vdots$		
$a_1 \; a_2 \; a_3 \; a_4 \cdots a_{n-1} \mid a_n$	1	$n-1$

BEST CASE # COMP $= n-1$

WORST CASE # COMP $= 1+2+3+\cdots+(n-1) = \dfrac{n(n-1)}{2}$

	BEST	AVG.	WORST
SELECTION	$\frac{n(n-1)}{2}$	$\frac{n(n-1)}{2}$	$\frac{n(n-1)}{2}$
BUBBLE	"	"	"
INSERTION	$n-1$	? (lab 3)	$\frac{n(n-1)}{2}$

IN ALL CASES: WORST $= \frac{n(n-1)}{2} = \frac{1}{2}n^2 - \frac{1}{2}n$

IF WE COUNTED PERIPHERAL OPS., WE WOULD GET

$$an^2 + bn + c$$

FOR SOME CONSTANTS a, b, c WHICH ARE MACHINE DEPENDENT.

GOAL: CONCENTRATE ON "n^2"

INFORMAL DEFINITION

$$f(n) = \Theta(g(n)) \qquad \text{"ORDER } g(n)\text{"}$$

IF $f(n) = \text{Const} \cdot g(n) + (\text{lower order terms})(n)$

i.e. lower order than $g(n)$ means:

$$\frac{L.O.T(n)}{g(n)} \to 0 \quad \text{AS } n \to \infty$$

Ex.

$$2n^2 + \underbrace{3n + 5}_{L.O.T(n)} = \Theta(n^2)$$

Const: 2, $g(n) = n^2$, L.O.T(n): $3n + 5$, $g(n) = n^2$

$$\frac{3n+5}{n^2} = \frac{3n}{n^2} + \frac{5}{n^2} = \frac{3}{n} + \frac{5}{n^2} \longrightarrow 0 \text{ AS } n \to \infty$$

$\frac{3}{n} \to 0$, $\frac{5}{n^2} \to 0$

Ex.

$$5n^3 + \underbrace{n + 100}_{LOT(n)} = \Theta(n^3)$$

Const: 5, $g(n) = n^3$, LOT(n): $n + 100$, $g(n) = n^3$

$$\frac{n+100}{n^3} = \frac{1}{n^2} + \frac{100}{n^3} \longrightarrow 0 \text{ AS } n \to \infty$$

$\frac{1}{n^2} \to 0$, $\frac{100}{n^3} \to 0$

<u>Ex.</u> $3\sqrt{n} + 2\sqrt[3]{n} + \sqrt[4]{n} = \Theta(n^{1/2})$

$$\underbrace{3}_{\text{const}}\ \underbrace{n^{1/2}}_{g(n)} + \underbrace{2n^{1/3} + n^{1/4}}_{L.O.T(n)} = \Theta(\underbrace{n^{1/2}}_{g(n)})$$

$$\frac{2n^{1/3} + n^{1/4}}{n^{1/2}} =$$

$$= \frac{2}{n^{1/2 - 1/3}} + \frac{1}{n^{1/2 - 1/4}}$$

$$= \underbrace{\frac{2}{n^{1/6}}}_{\to 0} + \underbrace{\frac{1}{n^{1/4}}}_{\to 0} \longrightarrow 0$$

THUS

	BEST	AVG	WORST
SELECTION	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$
BUBBLE	"	"	"
INSERTION	$\Theta(n)$	?	$\Theta(n^2)$

ASYMPTOTIC RUN TIME OF AN ALGORITHM

THIS IS A MACHINE INDEPENDENT MEASURE OF RUN TIME

Binary Search

INPUT. $n \geq 1$ (# list elements)

$a_1, \ldots, a_n$ (list values, **SORTED**)

target

OUTPUT. 0 if target is not found, OR

AN index i where TARGET is found.