

### Problem 1 [25 points]

On the model of Algorithm 4.2 (page 17) of the CAV book (by Henzinger and Alur, see link on web page), write an algorithm that performs the existential quantification of a variable. The algorithm takes a BDD  $B$  and an index  $i$ , and returns a BDD  $C$  with  $r(C) = \exists x_i.r(B)$ .

### Problem 2 [25 points]

Recall that if  $B$  is a propositional formula (represented as a BDD) defining a set of states, then  $\text{Post}(B)$  is the BDD denoting the set of successors of  $B$ . In formulas, if  $C = \text{Post}(B)$  then

$$C' = \exists \mathcal{V}.(B \wedge \mathcal{T})$$

where  $\mathcal{V}$  is the set of all variables, and  $\mathcal{T}$  is the transition relation.

- **Part 1 [10 points]**. Write an algorithm that, given BDDs  $A, B, C, D$ , checks whether there is a path that goes from  $A$  to  $D$  by visiting a state of  $B$  or  $C$ . Note that visiting a state of both  $B$  and  $C$ , while going from  $A$  to  $D$ , is fine.
- **Part 2 [15 points]**. Write an algorithm that, given BDDs  $A, B, C, D$ , checks whether there is a path that goes from  $A$  to  $D$  by visiting a state of either  $B$  or  $C$ , *but not both*. In other words, we want to know whether from  $A$  we can reach  $D$  by visiting one, and only one, of  $B$  and  $C$ .

### 1 Problem 3 [25 points]

Do Exercise 4.10 (page 9) of the CAV book (and comment on the time complexities).

### 2 Problem 4 [25 points]

Do Exercise 4.16 (page 13) of the CAV book.

### 3 Problem 5 [25 points]

*Extra credit problem.* Do Exercise 4.13 (page 12) of the CAV book. Give a proof of the size bounds for the BDDs.