

CMPE 177
Applied Graph Theory and Algorithms
Summer 2009

Midterm 1 Review Problems

1. Determine whether or not each of the following sequences is the degree sequence of a connected simple graph. Either draw a graph with the given degree sequence, or prove that none exists.
 - a. 1 2 3 4 4
 - b. 1 2 3 3 3
2. Let G be a simple graph on $n \geq 2$ vertices. Prove that G must contain two or more vertices of the same degree.
3. Let $G = (V, E)$ be a connected graph and $e \in E$. Prove that e is a bridge if and only if e belongs to every spanning tree of G .
4. Let G be a simple graph which is disconnected. Show that the complement of G is connected. (Hint: let x and y be any two vertices in \overline{G} and consider two cases: (1) x and y belong to distinct components of G , and (2) x and y belong to the same component of G .)
5. Let G be a graph with n vertices, m edges, and k connected components. Use induction on m to prove that $m \geq n - k$.
6. Let G be a connected graph and consider the following algorithm.
 1. While there remains a cycle in G
 2. Remove an edge from such a cycle

Prove that when this algorithm is complete, the resulting subgraph forms a spanning tree in G .