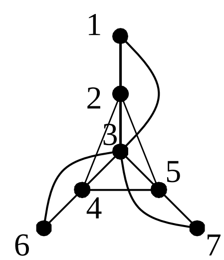
CMPE 177
Applied Graph Theory and Algorithms
Summer 2009

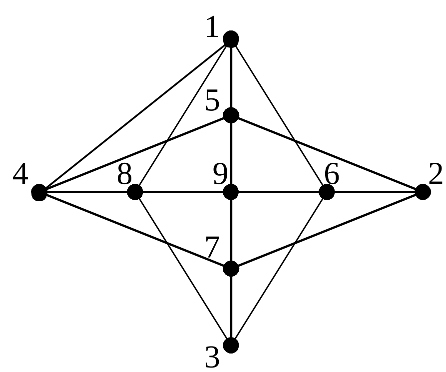
Final Review Problems

1. Determine whether the following graphs are Hamiltonian. Either give a Hamiltonian cycle or prove that none exists.

   (a)



   (b)



2. Prove that any connected graph on n vertices and m edges satisfies $m \geq n-1$. (Hint: induction on m.)

3. Let G be a graph on n vertices, m edges, and k connected components. Prove that if $m = n - k$, then G is acyclic. (Hint: Apply the result of the previous problem to each component of G, and use the treeness theorem.)

4. Let G be a connected graph with at least three vertices. Prove that if G has a bridge, then G has a cut vertex.

5. Recall a simple graph is called *self-complementary* if it is isomorphic to it's own complement. Let G be a simple graph on n vertices. Prove that if G is self-complementary, then either $n = 4t$, or $n = 4t + 1$ for some integer t.

6. Let G be a simple connected k-regular graph with $n = 2k - 1$ vertices. Prove that G is Hamiltonian. (Hint: use Dirac's theorem.)

7. Let f be a flow in a network N and let $A(X, \overline{X})$ be a cut in N.

   a. Prove that if $f(a) = c(a)$ for every $a \in A(X, \overline{X})$, and $f(a) = 0$ for every $a \in A(\overline{X}, X)$, then f is a maximum flow and $A(X, \overline{X})$ is a minimum cut.

   b. Conversely, prove that if f is a maximum flow in N and $A(X, \overline{X})$ is a minimum cut, then $f(a) = c(a)$ for every $a \in A(X, \overline{X})$, and $f(a) = 0$ for every $a \in A(\overline{X}, X)$.

8. Let f_1 and f_2 be flows in a network N and let $A(X, \overline{X})$ be a cut in N.

   a. Show that if f_1 and f_2 are both maximum flows then we need not have $f_1(a) = f_2(a)$ for every $a \in A(X, \overline{X})$ and every $a \in A(\overline{X}, X)$.

   b. If f_1 and f_2 agree on both $A(X, \overline{X})$ and $A(\overline{X}, X)$, are both f_1 and f_2 maximum flows?

9. Draw two non-isomorphic strongly connected digraphs having indegrees {1, 1, 1, 2, 2, 2} and outdegrees {1, 1, 1, 2, 2, 2}. (One word of clarification: the ordering in the preceding sets is irrelevant, i.e. a vertex of indegree 1 may have outdegree 1 or outdegree 2.) Prove that your two digraphs are non-isomorphic.

10. Let T be a tournament on n vertices (i.e. a digraph whose underlying graph is K_n). Suppose that all outdegrees in T are equal. Prove that n is odd.