**Basic concepts in signal processing**
Audio, images, and video are 1D, 2D, and 3D signal. Some basic knowledge of signal process will help us to understand multimedia processing
**Continuous signal**
1-dimensional signal can be represented as a function (voltage, intensity, pressure, etc)

$$f(t)$$

Domain of the function: time. The general case $t \in [-\infty, \infty]$
The impulse (delta, Dirac) function $\delta(t)$
Limit of a square function $\lim_{\varepsilon \to 0} f(x) = \begin{cases} 1/\varepsilon & x \in [-\varepsilon/2, \varepsilon/2] \\ 0 & otherwise \end{cases}$

$\int_{-\infty}^{\infty} x(t)dt = 1$

**Discrete signal**
Sampled version of the continuous function, denoted as

$$f(n)$$

Where $n \in \mathbf{Z}$

**Convolution**

$$y(t) = x(t) \otimes h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$y(n) = x(n) \otimes h(n) \sum_{-\infty}^{\infty} x(k)h(n-k)$$

Example (Chalkboard)

Properties of convolution
Commutative
$x(t) \otimes h(t) = h(t) \otimes x(t)$
Associative
$(x(t) \otimes y(t)) \otimes z(t) = x(t) \otimes (y(t) \otimes z(t))$
Distributive
$x(t) \otimes (y(t) + z(t)) = x(t) \otimes y(t) + x(t) \otimes z(t)$
Impulse function
$x(t) \otimes \delta(t) = x(t)$

**Fourier transform of a continuous signal**
Decompose a continuous signal into signals of various frequencies

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

Interpretation:
$e^{-j\omega t} = \cos(\omega t) - j\sin(\omega t)$
$F(\omega)$ represents the amount of the signal in angular or radial frequency $\omega = 2\pi f$

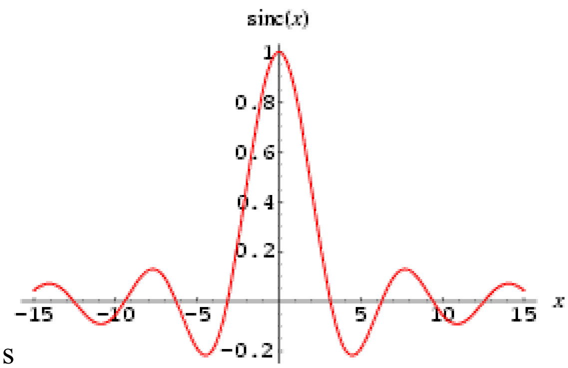
**Inverse Fourier transform**

$$F(t) = \int_{-\infty}^{\infty} f(\omega) e^{j\omega t} d\omega$$

**The FFTs of some typical functions**

Sinc function f(x)=sinx/x

Rect(x)={1 if –1/2<x<1/2}

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See table:

**Properties of FFT**

See attached table

**Discrete Fourier Transform**

$$S(k) = \sum_{n=0}^{N-1} s(n) e^{-j\frac{2\pi nk}{N}}$$

$$s(n) = \frac{1}{N} \sum_{k=0}^{N-1} s(k) e^{j\frac{2\pi nk}{N}}$$

**Sampling Theory**

Nyquist theorem

If f(t) is bandlimited to $[-\omega_B, \omega_B]$, we can reconstruct it **perfectly** from its samples

$$f_s[n] = f(nT)$$

for $\omega_s = 2\pi / T > 2\omega_B$

**Filtering**

Filtering is convolution = Frequency multiplication

Perfect low pass filtering: multiplied by a box function in frequency domain is equivalent to convolution with a sinc function in the time domain.

Filter design:
(1) Find the frequency response of the filter.
(2) Inverse DFT transform the filter