—CHAPTER 6 —

DRAWING A FREEBODY DIAGRAM

Consider the two friends leaning against one another in **Figure 6.1A**. The freebody diagram of one leaning friend is shown in **Figure 6.1C**. In going from 6.1A to 6.1C, we zoomed-in and drew a boundary around the system (friend 1) to isolate him from his surroundings, as shown in **Figure 6.1B**. This boundary is an imaginary surface and the system is (by definition), the "stuff" inside this imaginary surface. The rest of the world is everything else. You can think of the boundary as a shrink wrap around the system.

After drawing the boundary, we identified the external loads acting on the system at or across this boundary and drew them on the isolated system at their points of application. These loads represent how the surroundings push, shove, and twist the system. In a freebody diagram we draw the system (the "stuff" inside the imaginary surface) and replace the surroundings with the loads these surroundings apply to the system. It is important to recognize that we are not ignoring the surroundings---we simply replace them with the loads that the system experiences because of them.

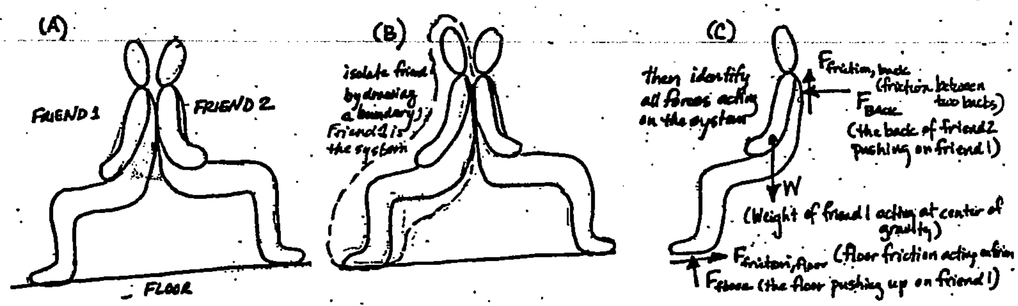
A freebody diagram is a drawing of a system and the loads acting on it and is perhaps the most important tool in this book. Creating a freebody diagram involves separating the portion of the world that you're interested in (the system) from its surroundings (the rest of the world), then drawing the system. Next all the loads (forces and couples) acting on the system are identified and added to the drawing.

This chapter is devoted solely to creating freebody diagrams. We build on the work in prior chapters and on the Analysis Procedure presented in Chapter 1. Creating a freebody diagram is part of the DRAW step in the Analysis Procedure. We can use the freebody diagram to write the equations of equilibrium, which are in turn used to determine the relationship between loads (but we are getting ahead of ourselves; that's covered in the next chapter!).

6.1 Types of External Loads

Some of the loads acting on a system act *across* the boundary of the system; the principle example of this type of load is **gravity** (which manifests itself as weight). Another is magnetic force, which results from electron-magnetic field interaction. Other loads act *directly on* the boundary and are caused by:

FIG 6.1



(A)
FRIEND 1
FRIEND 2
FLOOR
(B)
isolate friend by drawing a boundary; Friend 1 is the system
(C)
then identify all forces acting on the system
$F_{friction, back}$ (friction between two backs)
F_{BACK} (the back of friend 2 pushing on friend 1)
W
(Weight of friend 1 acting at center of gravity)
$F_{friction, floor}$ (floor friction acting on him
F_{floor} (the floor pushing up on friend 1)

• solid contact between the system and the rest of the world. These loads, called **solid boundary conditions**, are either some physical connection (e.g., a bolt, wire, or weld) or simply where the system rests against the rest of the world. Solid boundary conditions are comprised of the contact and bonding forces of normal contact, friction, tension, compression, and shear that we introduced in Chapter 4.

• fluid pressing on the boundary. These loads, called **fluid boundary conditions**, are either a pressure (force per area) on the boundary and/or forces tangent to the surface.

In practice, a system may be loaded by a combination of gravity, solid boundary conditions and fluid boundary conditions, as illustrated in **Figure 6.2.** Notice that at some boundary locations no loads act. At other locations there are so-called **known loads**—for example, in Figure 6.2, a 40 kN gravity force acts on the boat, and a 20 N horizontal force results from the hand pulling on the lever).

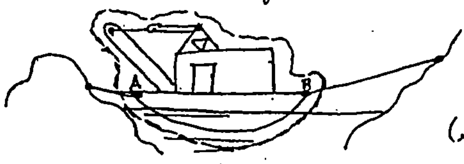
Regardless of the nature of a load acting on a system, when that load is drawn on a freebody diagram it is represented by a vector or a distribution of vectors acting at a point of application. It is given a unique variable label (e.g., the force acting at point A in Figure 6.2B in the x direction is labeled $\mathbf{F_{Ax}}$), and its magnitude (if known) is written next to the vector.

6.2 PLANAR AND NON-PLANAR SYSTEMS

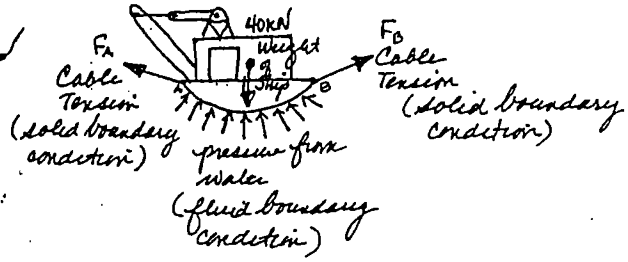
Defining the loads at boundary conditions is easier if we can classify the system as a **planar system**, which is a system where all the loads (forces and couples) acting on the system can *reasonably be assumed* to lie in a single plane. This is generally the case when known loads, gravity loads, and the points of application associated with solid and fluid boundary conditions are all in a single plane. **Figure 6.3A** shows an example of a planar system; it is possible to classify this system as planar because the gravity force, cable tension, and pin-connection are all in a single plane. Sometimes planar systems are referred to as two-dimensional (2D) systems. The freebody diagram associated with a planar system typically requires only a single plan view of the system (reference back to Chapter 1 here).

A system with a plane of symmetry in regards to both its geometry and the loads acting on it may also be treated as a planar system. A plane of symmetry is a plane such that the

(A) Physical situation with
Boundary shown that
isolates the system (the ship)

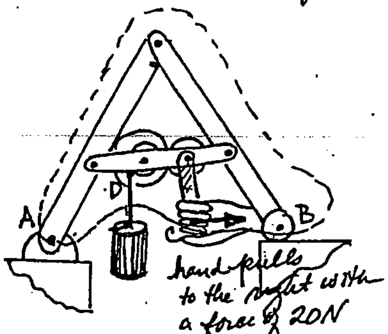


Freebody diagram of the Ship



(B)

Physical situation with boundary shown that isolates the system (the frame)



Freebody diagram of the frame.

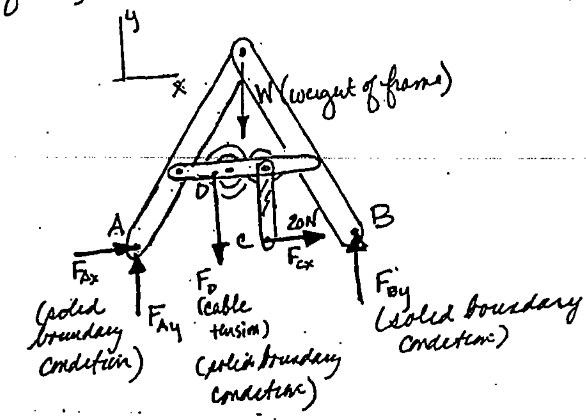


FIG 6.2

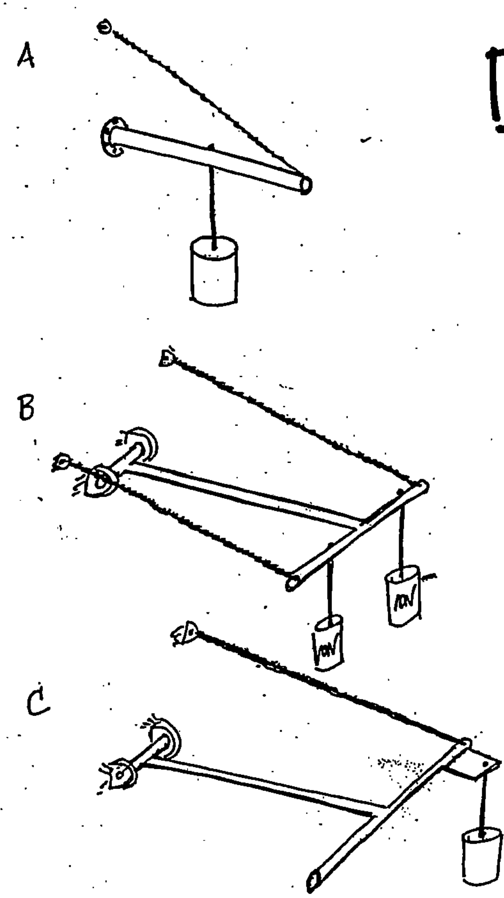


FIG 6.3

portion of the system on one side of the plane is a mirror image of the portion that is on the other side. **Figure 6.3B** illustrates a system with a plane of symmetry and how it can be treated as a planar system.

If it is not possible to define a single plane such that all forces and couples are in this plane, or there is no plane of symmetry, the system is classified as a **non-planar system**. **Figure 6.3C** shows an example of a non-planar system; it is not possible to define a plane that contains the gravity force, cable tension and pin-connection. Sometimes non-planar systems are referred to as three-dimensional (3D) systems. The freebody diagram associated with a non-planar system typically requires an isometric drawing or multiple plan views.

Sample Problem #1 Identifying planar and non-planar systems

6.3 SOLID BOUNDARY CONDITIONS—PLANAR SYSTEMS

Now we look at how to identify and draw the loads acting at solid boundary conditions associated with planar systems. Consider the system in **Figure 6.4A** for which we want to draw a freebody diagram. At the boundary we identify:

- simple contact without friction (at A),
- a cable attached to the system (at B),
- a spring attached to the system (at C),
- contact with friction (at D),
- the system fixed to its surroundings (at E)
- the system pinned to its surroundings (at F),
- a link attached to the system (at G),
- a force of known direction and magnitude (at H),
- a couple of known direction and magnitude (at I).

At each location we consider whether the rest of the world acts on the system with a force and/or a couple. As a general rule, if a boundary location prevents the translation of the system in a given direction, then a force acts on the system in the opposite direction. Likewise, if rotation is prevented, a couple opposite the rotation acts on the system.

At A (simple contact without friction) This solid boundary condition results from the system resting against a smooth, frictionless surface. Simple contact consists of a force acting on the system that is normal to the surface on which the system rests (see **Figure**

FIGURE 6.4

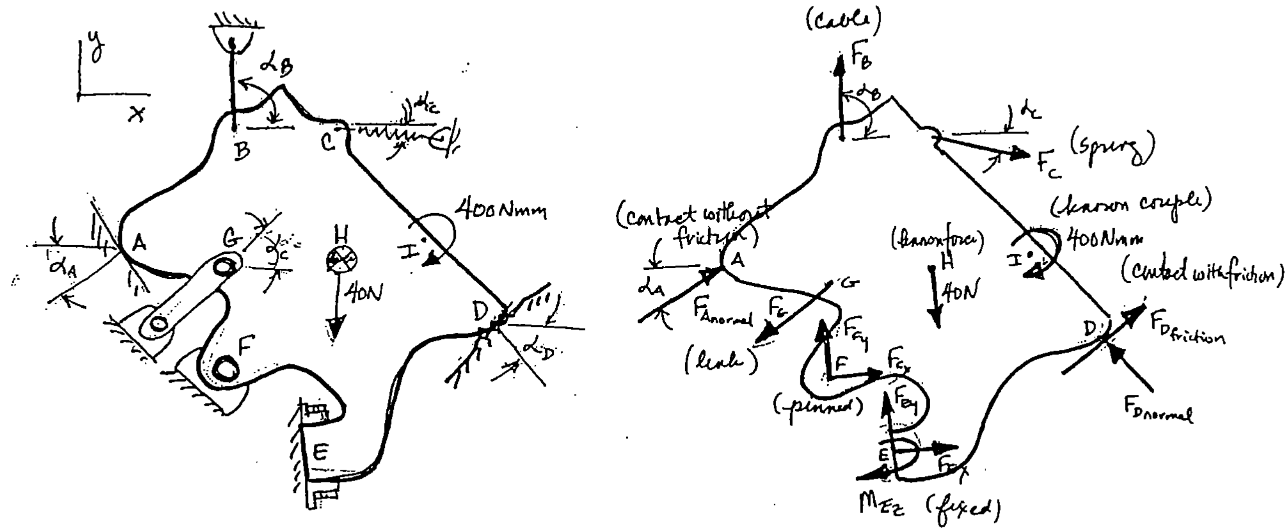
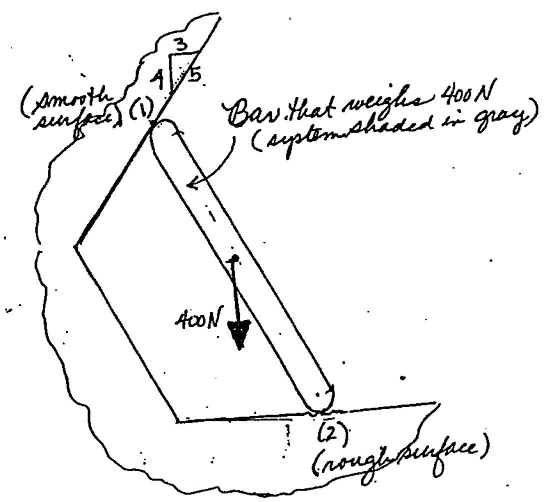
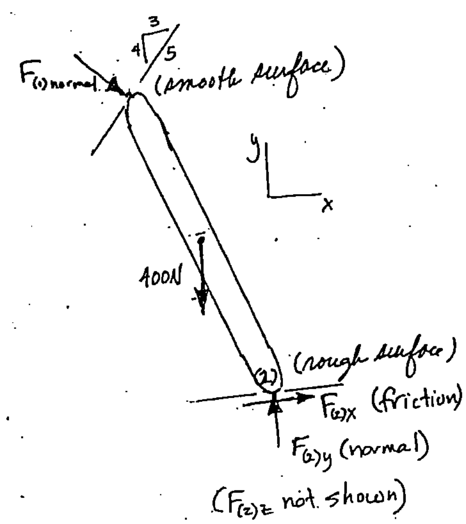


FIGURE 6.5

(A)



(B)



6.5A, upper surface). This force prevents the system from moving into the surface and is oriented so as to push on the system, as shown in **Figure 6.5B**. Since the upper surface is smooth, friction between the system and its surroundings is very small. Therefore, we choose to neglect this very small friction and assume that there is no force component tangent to the surface.

In **Figure 6.4B** (the freebody diagram of Figure 6.4A) the force resulting from simple contact at A is represented by $\mathbf{F_A}$; we know its direction is normal to the surface so as to *push* on the system.

At B (a cable) This solid boundary condition consists of a force acting on the system; its line of action is along the axis of the cable. The force represents the cable pulling on the system since the cable is in tension, as illustrated in **Figure 6.6.**

In **Figure 6.4B** the force from the cable at B is represented by $\mathbf{F_B}$; we know its direction is along the cable axis, so as to *pull* on the system.

At C (a spring) This solid boundary condition consists of a force that pushes or pulls on the system; its line of action is along the axis of the spring. If the spring is extended by an amount Δ, the spring is in tension and the force is oriented so as to pull on the system **(Figure 6.7A)**. If the spring is compressed by an amount Δ, the force is oriented so as to push on the system (**Figure 6.7B**). The magnitude of the force is proportional to the amount of spring extension or compression, and the proportionality constant is known as the spring constant, "k." In other words, the magnitude of the force is equal to the product of "k" and the spring extension or compression:

$$F_c = k(\Delta)$$

where k is the spring constant in units of force/length (e.g., N/mm).

In **Figure 6.4B** the spring force at C is represented by $\mathbf{F_C}$; we know its direction is along the spring axis. Furthermore, if the spring is in tension (compression), the force will act so as to *pull (push)* on the system.

At D (contact with friction). This solid boundary condition consists of two forces. One of these is a normal force (just like simple contact). The second force is due to friction and is

FIGURE 6.6

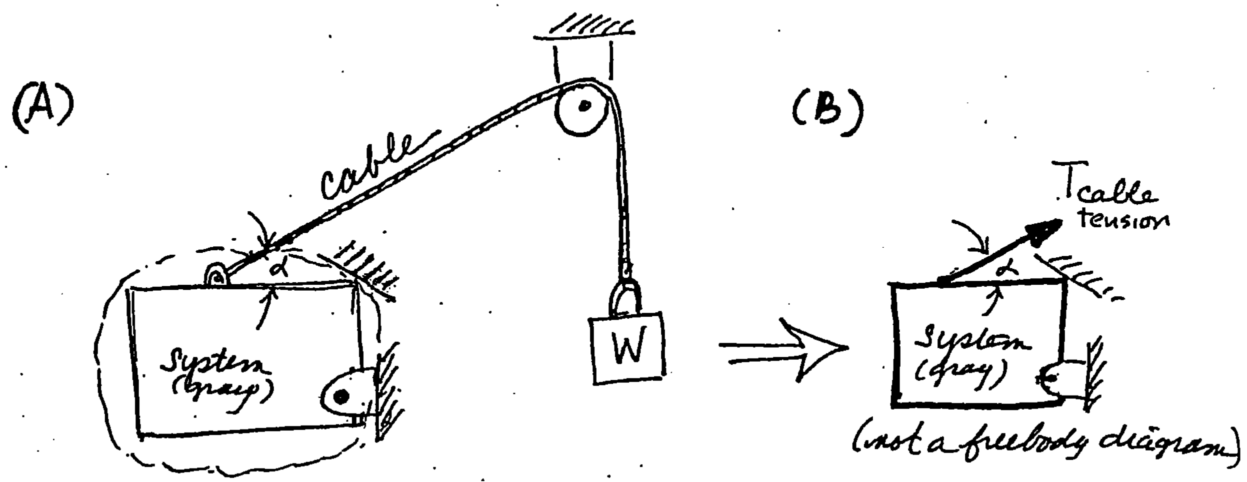
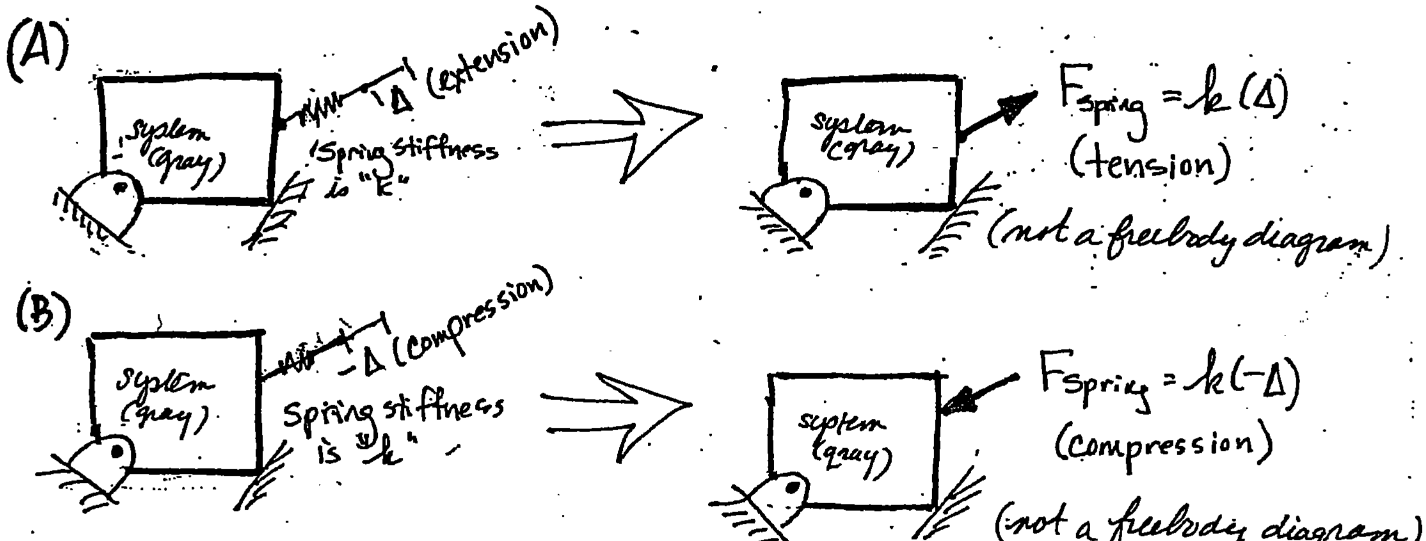


FIGURE 6.7



tangent to the surface against which the system rests---therefore it is perpendicular to the normal force. Contact with friction is illustrated in the lower surface contact in **Figure 6.5A and B.**

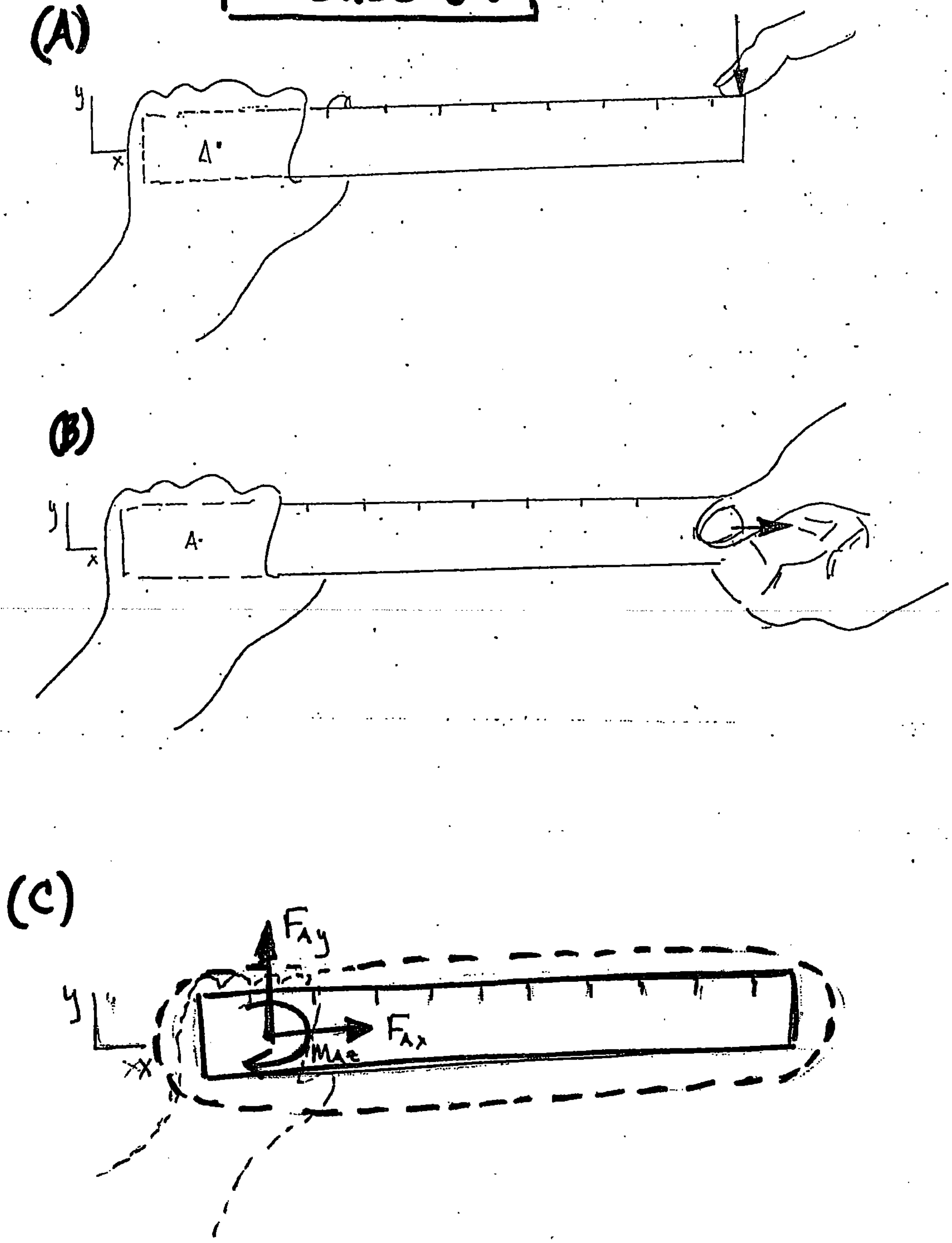
The force due to friction ($\mathbf{F}_{friction}$) is related to and limited by normal contact force ($\mathbf{F}_{normal}$) and the characteristics (e.g., smoothness) of the contact. Often the relationship between $\mathbf{F}_{friction}$ and $\mathbf{F}_{normal}$ is represented in terms of the Coulumb Friction Model (reference here). This model states that if $F_{friction} < \mu_{static} F_{normal}$ there will no sliding of the system relative to its surroundings, where μ_{static} is the static coefficient of friction and ranges from 0.01-0.500 (unitless), depending on the characteristics of the contact. If, on the other hand, $F_{friction} = \mu_{static} F_{normal}$, there will be sliding of the system relative to its surroundings.

In **Figure 6.4B** the normal force at D is represented by $\mathbf{F}_{Dy}$; we know its direction is normal to the surface (in the y direction) so as to push on the system. The friction force is represented by $\mathbf{F}_{Dx}$ and is perpendicular to the normal force (we have arbitrarily drawn it in the positive x direction).

At E (system fixed to its surroundings, referred to as a fixed condition) This solid boundary condition consists of a force and a couple. To get a feeling for a fixed condition, consider the set-up shown in **Figure 6.8** (better yet, reproduce it yourself). The ruler is the system and your hands are the surroundings. With your left hand, firmly grip one end of the ruler, and with your right hand, apply a force, as shown in **Figure 6.8A**. Notice that your left hand automatically applies loads to the ruler in order to keep the gripped end from moving; these loads "fix" the gripped end relative to your hand. Try applying other loads to the ruler (see **Figures Figures 6.8B and 6.8C**) —notice how your left-hand applies appropriate loads so that the gripped end remains fixed. Your left-hand is able to apply loads (forces and couples) to the ruler at the gripped end so that the ruler remains fixed. These loads can be represented as a force of $\mathbf{F}_{left\text{-}hand} = F_{lefthand,x}\mathbf{i} + F_{lefthand,y}\mathbf{j}$ and a couple (pure moment) as $\mathbf{M}_{left\text{-}hand} = M_{lefthand,z}\mathbf{k}$ and are the net effect of your left-hand gripping the ruler.

Now returning to the system depicted in **Figure 6.4A**, we can describe the loads acting at the fixed condition at E as $\mathbf{F}_E = F_{Ex}\mathbf{i} + F_{Ey}\mathbf{j}$ and the couple as $\mathbf{M}_E = M_{Ez}\mathbf{k}$. These loads are shown in **Figure 6.4B.**

FIGURE 6.8



At F (the system is pinned to its surroundings, referred to as a pin joint). A pin joint is comprised of a pin that is loosely fitted in a hole. This solid boundary condition consists of a force. To get a feeling for the force at a pin joint consider the physical set-up in **Figure 6.9A.** The ruler (which is the system) is lying on a flat surface. A pencil, which is acting like a pin, is placed in the hole in the ruler and is gripped firmly with your left-hand. The pencil and your hands comprise the surroundings. Now load the system with your right-hand, as shown in **Figure 6.9A**; notice how your left hand reacts with a force to counter the right hand force. Also orient the load as shown in **Figure 6.9B**; again your left hand counters with a force. Finally, load the ruler as shown in **Figure 6.9C**, and notice that the ruler rotates because your left hand is unable to counter with a pure moment. We have just demonstrated that there is a force acting on the system ($\mathbf{F}_{left\text{-}hand}$) at the point-joint but there is no couple. The force $\mathbf{F}_{left\text{-}hand}$ lies in the plane perpendicular to the pencil (pin) axis. For the situation in **Figure 6.9**, this means that $\mathbf{F}_{left\text{-}hand}$ can be written as $\mathbf{F}_{left\text{-}hand}=F_{lefthand,x}\mathbf{i}+F_{lefthand,y}\mathbf{j}$.

Now returning to the system depicted in **Figure 6.4A**, we can describe the load acting at the pin joint at F as $\mathbf{F}_F=F_{Fx}\mathbf{i}+F_{Fy}\mathbf{j}$, as shown in **Figure 6.4B.**

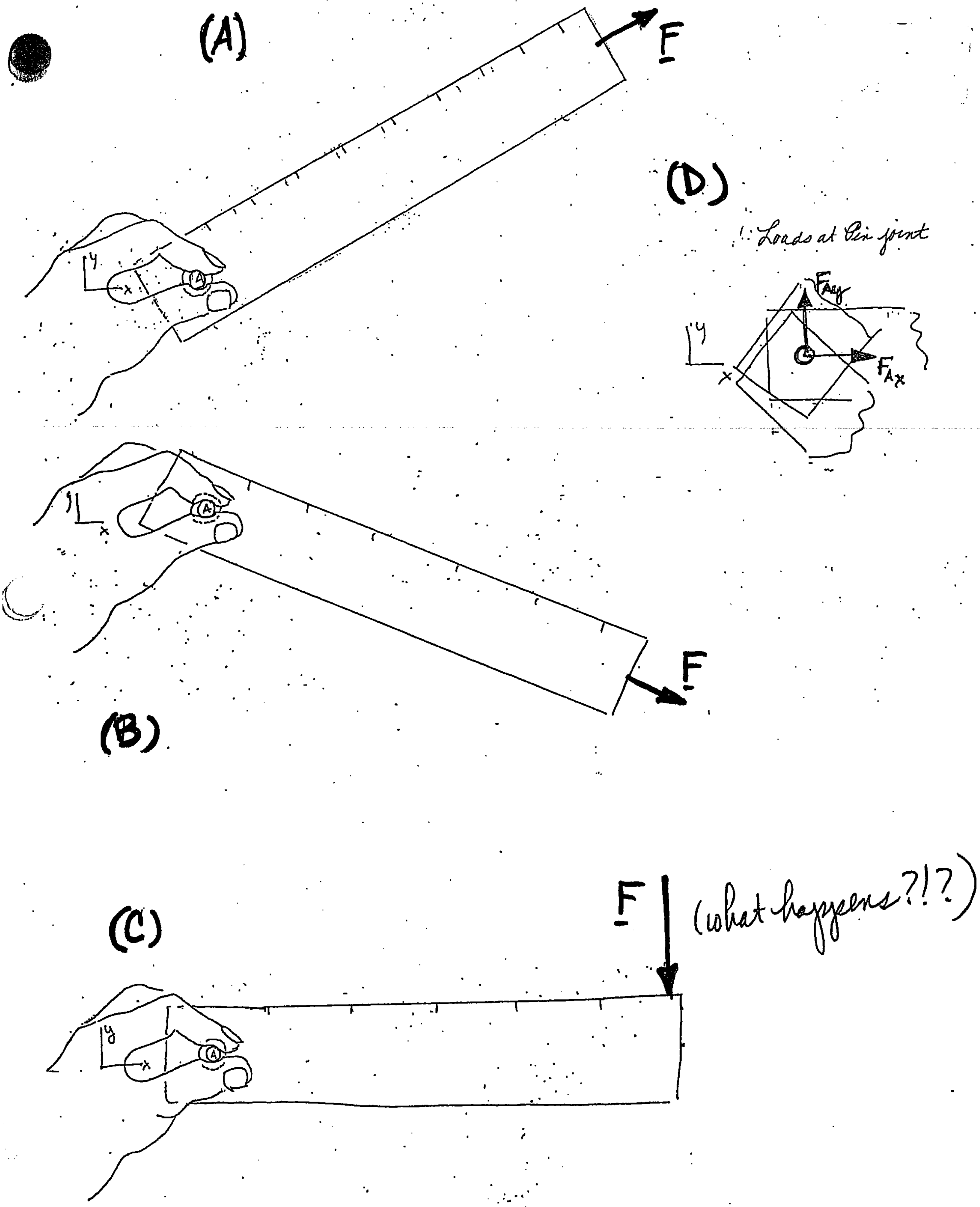
At G (a link is attached to the system) This solid boundary condition consists of a force that pushes or pulls on the system; its line of action is along the axis of the link. We will have a lot more to say about links in the next chapter--- for now we simply say that a link is a member with a pin joint at each end and no other loads acting on it.

In **Figure 6.4B** the force at G is represented by $\mathbf{F}_G$, acting along the axis of the link. Since a link may either *push or pull* on the system, we have arbitrarily chosen to orient it with the positive y axis.

At H and I known loads act. These loads are the surroundings acting on the system due to normal contact, friction, tension, compression and/or shear forces. What differentiates these loads from those associated with other solid boundary conditions is that we generally know both their direction and magnitude. Sometimes these known loads are referred to as the **design loads**, as they are the loads that the system is intended to stand-up to.

Now returning to the system depicted in **Figure 6.4A**, we can describe the loads acting at G as $\mathbf{F}_H=(10\ \mathbf{i}+20\ \mathbf{j})$ N and the couple as $\mathbf{M}_E=(500\ \mathbf{k})$ Nm. These loads are shown in **Figure 6.4B.**

FIGURE 6.9



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The freebody diagram of the planar system in **Figure 6.4A** is presented in **Figure 6.4B**. It includes loads due to solid boundary conditions, as well as the load due to gravity.

Table 6.1 summarizes the solid boundary conditions discussed above. Other commonly found solid boundary conditions are also included in the table. Don't feel that you need to memorize all of the conditions in this table---it is presented as a "ready reference." On the other hand, you should be familiar with these standard boundary conditions and the loads that they represent.

Sample Problem #2 Complete freebody diagrams in Figures 6.5 and 6.6 using Table 6.1

Sample Problem #3 Evaluating the correctness of freebody diagrams

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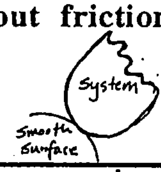
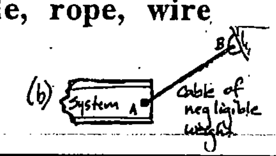
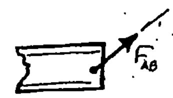
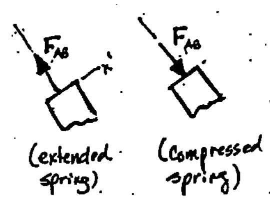
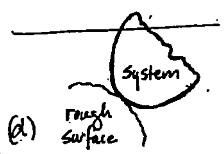
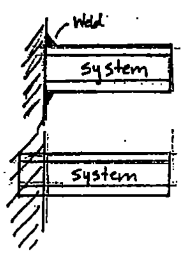
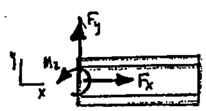
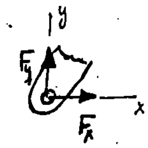
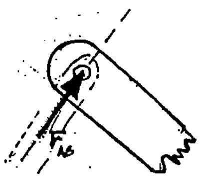
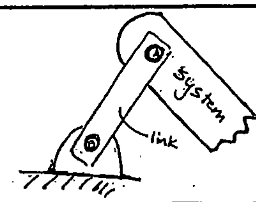
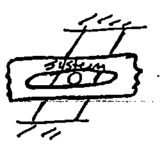
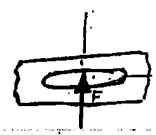
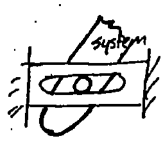
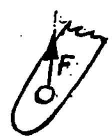
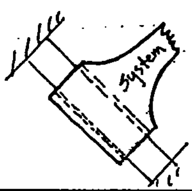
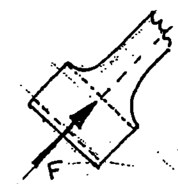
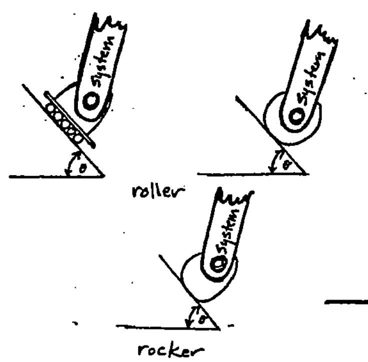
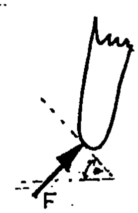
Table 6.1 Standard Solid Boundary Conditions for Planar Systems

(A) **Type of Solid Boundary Condition** (each cell below will have a sketch of the connection, including symbol used in text for denoting this connection. Some cells will also have small photos)	**Description of Boundary Condition Loads**	(B) **Loads**
(1.) Simple contact (without friction) 	**force** oriented normal to surface on which system rests. Direction is such that it pushes on system.	**F** 
(2.) Cable, rope, wire 	**force** oriented along the axis of cable. Direction is such that it pulls on system.	**F** 
(3.) Spring 	**force** oriented along the axis of spring. Direction is such that it pulls on system if spring is in tension,and pushes if spring is in compression.	**F** 
(4.) Contact with friction 	**two forces**, one oriented normal to surface so as to push on system (F_y). Other force (F_x) is tangent to the surface on which the system rests.	F_y F_x 
(5.) Fixed condition 	**force** in xy plane of unknown direction and magnitude. Represented as x and y components. **couple (pure moment)** about z axis of unknown magnitude	$F_x + F_y$ M_z 

(6.) Pin joint (pin or hole part of system)  	**force** in plane perpendicular to pin axis. Point of application is at center of pin. Orientation within plane unknown, so force is represented as x and y components.	**$F_x + F_y$** 
(7.) Link 	**force** oriented along the axis of link. A link can push or pull on the system. Magnitude unknown.	**F** 
(8.) Pin-in-slot (slot part of system)  	**force** oriented normal to the axis of the slot. Direction is such that it can pull or push on the system	**F** 
(9.) Slot-on-pin (pin part of system)  	**force** oriented normal to the axis of the slot. Direction is such that it can pull or push on the system	**F** 
(10.) Collar on shaft  	**force** oriented normal to the axis of the shaft. Direction is such that it can pull or push on the system	**F** 
(11.) Roller or Rocker  	**force** oriented normal to surface on which system rests. Direction is such that it pushes on system.	**F** 

6.4 SOLID BOUNDARY CONDITIONS—NON-PLANAR SYSTEMS

We now consider how to identify and draw the loads that makeup solid boundary conditions for non-planar systems. You will see similarities to our discussion in the prior section on planar systems AND some important differences. Like planar systems, *if a boundary location prevents the translation of the system in a given direction, then a force acts on the system in the opposite direction. Likewise, if rotation is prevented, a couple opposite the rotation acts on the system.*

Consider the system in **Figure 6.10A** for which we want to draw a freebody diagram. This system is non-planar because all the points of application of solid boundary conditions do not lie in the plane defined by the known forces. The first four solid boundary conditions acting on this non-planar system (1. **simple contact without friction**, 2. **cable**, 3. **link**, 4. **spring**) are identical to their planar counterparts. Associated with each of the boundary conditions is a force acting with a known line-of-action, as depicted in **Figure 6.10B**.

The fifth (5. **contact with friction**) and the sixth (6. **fixed**) solid boundary conditions are similar to their planar counterparts. **Contact with friction** involves normal ($\mathbf{F}_{5,normal}$) and friction ($\mathbf{F}_{5,friction}= F_{5x}\mathbf{i}+F_{5y}\mathbf{j}$) forces, where the friction force is perpendicular to the normal force. The **fixed** condition acting on a non-planar system is able to prevent the system from translating along and rotating about any axis---therefore it involves a force ($\mathbf{F}_6=F_{6x}\mathbf{i}+F_{6y}\mathbf{j} + F_{6y}\mathbf{k}$) and a couple ($\mathbf{M}_6= M_{6x}\mathbf{i} + M_{6y}\mathbf{j} +M_{6z}\mathbf{k}$).

The seventh solid boundary condition is a (7.) **hinge**; it does not restrict rotation of the system about the hinge pin. If a hinge is one of several solid boundary conditions acting on a system, it applies a force to the system that is perpendicular to the pin axis. In contrast, if there is only a hinge solid boundary condition, the hinge will apply a force and a couple perpendicular to the pin axis to the system (see **Figure 6.11**). We refer to this solid boundary condition as a **single hinge**.

Since the hinge in **Figure 6.10A** is one of several solid boundary conditions acting on the system, it applies a force perpendicular to the pin axis and no couple, as illustrated in **Figure 6.10B**.

FIGURE 6.10

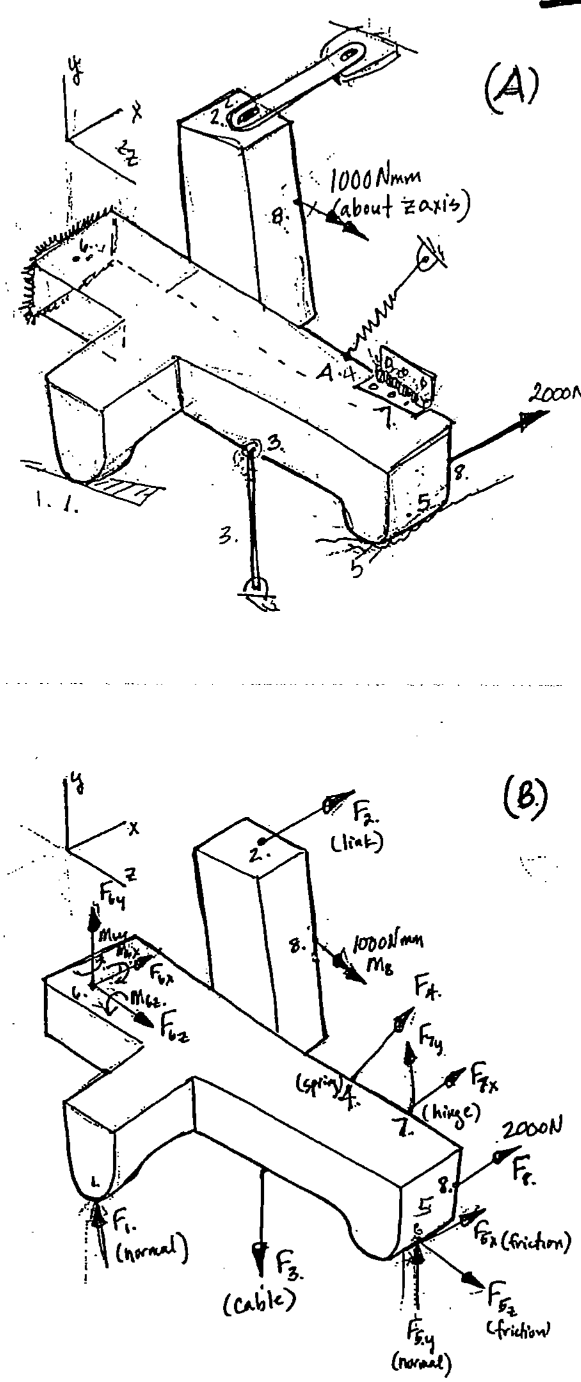


FIGURE 6.11

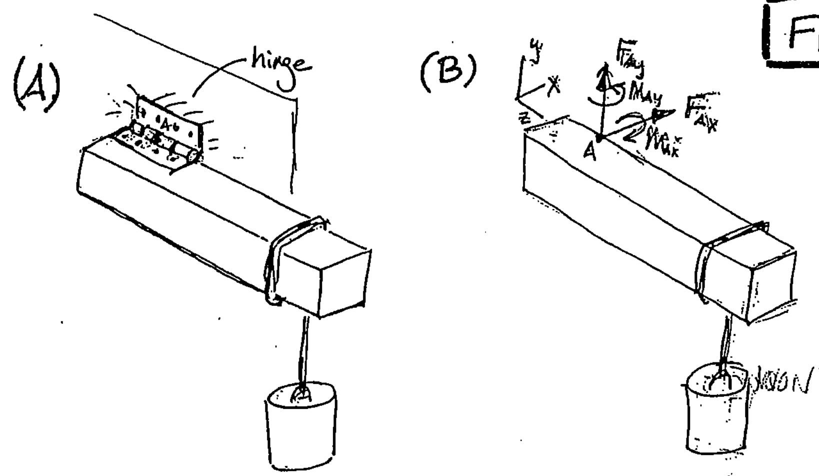


Figure 6.10B is the freebody diagram that represents the solid boundary conditions shown in Figure 6.10A in terms of loads (forces and/or couples). This diagram also includes the known loads.

Other solid boundary conditions commonly found with non-planar systems are included in Table 6.2. For example, the **ball-and-socket joint** restricts all translations of the system by applying a force to the system ($\mathbf{F}=F_x\mathbf{i}+F_y\mathbf{j}+F_y\mathbf{k}$). It does not restrict rotation of the system about any axis. An example of a ball-and-socket joint familiar to practically everyone is a human hip joint (**Figure 6.12**). Take a few minutes to study Table 6.2 and notice the similarities and differences between thrust bearings, journal bearings, and hinges. (If you are wondering why hinges and bearings apply different loads to a system depending on their number---hold that question. We will address it in the next chapter.)

Table 6.2 is not an exhaustive list of solid boundary conditions associated with non-planar systems. It contains commonly found and representative examples. If you find yourself considering a solid boundary condition that is not neatly classified as one of these, remember that you can always return to the basic characteristics associated with any solid boundary condition; namely *if a boundary location prevents the translation of the system in a given direction, then a force is exerted on (acts on) the system in the opposite direction. Likewise, if rotation is prevented, a couple opposite the rotation is exerted on the system.*

****

Sample Problem #4 Inspecting existing freebody diagrams for correctness

Sample Problem #5 Using questions to define loads at solid boundary conditions

FIGURE 6.12

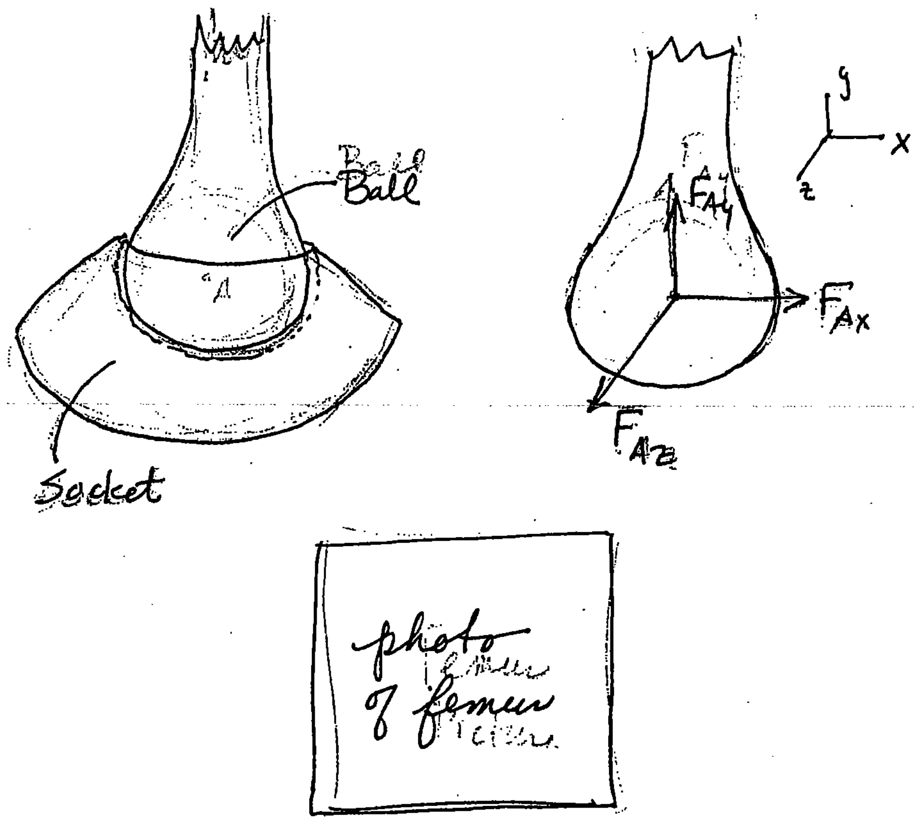
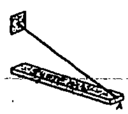
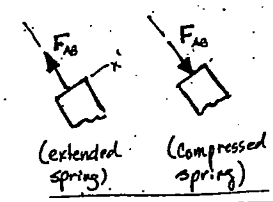
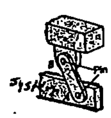
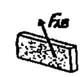
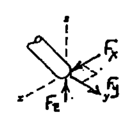
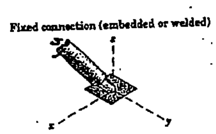
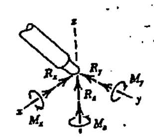
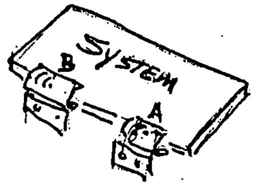
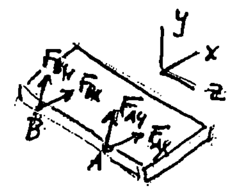
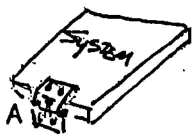
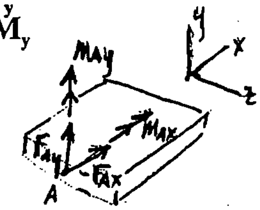
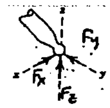
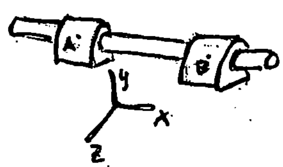
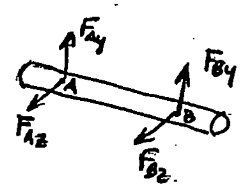
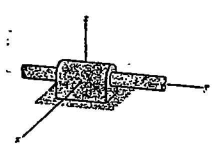
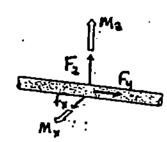
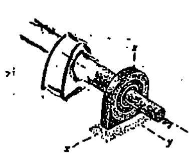
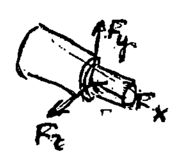
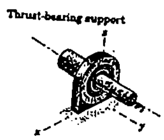
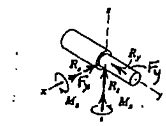
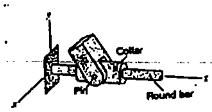
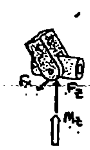
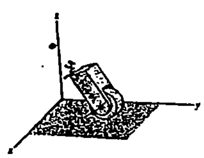
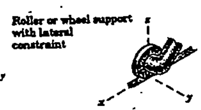
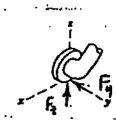


Table 6.2 Standard Solid Boundary Conditions for Non-Planar Systems

(A) **Type of Solid Boundary Condition** (each cell below will have a sketch of the connection, including symbol used in text for denoting this connection. Some cells will also have small photos)	**Description of Boundary Condition Loads**	(B) **Loads**
(1.) Simple Contact (without friction) 	**force** oriented normal to surface on which system rests. Direction is such that it pushes on system.	**F** 
(2.) Cable, rope, wire 	**force** oriented along the axis of cable. Direction is such that it pulls on system.	**F** 
(3.) Spring 	**force** oriented along the axis of spring. Direction is such that it pulls on system if spring is in tension and pushes if spring is in compression.	**F** 
(4.) Link 	**force** oriented along the axis of link. A link can push or pull on system. Magnitude unknown.	**F** 
(5.) Contact with Friction 	**two forces**, one oriented normal to surface so as to push on system (F_y). Other force (F_x+ F_z) is tangent to the surface on which the system rests.	F_y F_x+ F_z 
(6.) Fixed condition 	**force** of unknown direction and magnitude. Represented by its x, y and z components. **couple (pure moment)** of unknown direction and magnitude, represented as its x, y and z components.	F_x+ F_y + F_z M_x + M_y+ M_z 

(7A.) Hinge 	**force** in plane perpendicular to shaft axis. Point of application is at center of shaft. Orientation within plane unknown, so force is represented by x and y components.	$\mathbf{F_x + F_y}$ 
(7B.) Single Hinge (consists of shaft and articulated collar) 	**force** in plane normal to shaft axis. Represented as: **couple (pure moment)**, with components about axes perpendicular to shaft axis.	$\mathbf{F_x + F_y}$ $\mathbf{M_x + M_y}$ 
(8.) Ball and socket (ball or socket part of svstem) 	**force** of unknown direction and magnitude.	$\mathbf{F_x + F_y + F_z}$ 
(9A.) Journal Bearing (shaft or sleeve part of system) (consists of a shaft running through a frictionless collar) 	**force** in plane perpendicular to shaft axis. Point of application at center of shaft. Orientation within plane unknown, so force represented as x and y components.	$\mathbf{F_x + F_y}$ 
(9B.) Single Journal Bearing 	**force** in plane normal to shaft axis. Represented as: **couple (pure moment)**, with components about axes perpendicular to shaft axis.	$\mathbf{F_x + F_z}$ $\mathbf{M_x + M_z}$ 
(10A.) Thrust Bearing 	**two forces**, one in plane perpendicular to shaft axis ($\mathbf{F_{x'} + F_{y'}}$). The other force is in the direction of shaft axis ($\mathbf{F_{z'}}$), and is sometimes referred to as the "thrust force". Point of application is at center of shaft.	$\mathbf{F_x + F_y + F_z}$ 

(10B.) Single Thrust Bearing (consists of a shaft running through a frictionless collar) 	**two forces**, one in plane perpendicular to shaft axis ($\mathbf{F_x + F_y}$.) The other force is in the direction of shaft axis ($\mathbf{F_z}$), and sometimes referred to as the "thrust force." Point of application is at center of shaft. **couple (pure moment)**, with components about axes perpendicular to shaft axis.	$\mathbf{F_x + F_z + F_z}$ $\mathbf{M_x + M_z}$ 
(11.) Collar on shaft with pin (collar or shaft part of system) 	**force** oriented normal to axis of shaft. Direction is such that it can pull or push on the system. **couple (pure moment)** about axis normal to plane.	$\mathbf{F_y}$ $\mathbf{M_x}$ 
(12.) Smooth Roller in Guide  	**two forces**, one oriented normal to point of contact on surface ($\mathbf{F_x}$). Other force normal to rolling-direction ($\mathbf{F_z}$).	$\mathbf{F_x + F_z}$  

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6.5 SOLID BOUNDARY CONDITIONS CONSISTING OF DISTRIBUTED FORCES

Up to this point we have modeled solid boundary conditions as loads acting at a single location on the system boundary. In actuality, all solid boundary conditions consist of forces distributed over a finite surface area—for example, if you press down on a table with your hand, the force you apply to the table is distributed over a finite area (**Figure 6.13A**). For many practical applications, we can "condense" or sum this distributed force into a single point force (**Figure 6.13B**). There are, however, solid boundary conditions comprised of distributed forces where we explicitly consider the loads to be distributed; **Figure 6.14** shows some examples.

The key idea that we want to get across is that solid boundary conditions consisting of distributed forces must be included in the system's freebody diagram. These forces can be represented in the diagram as distributed forces or as a single net force (see **Figure 6.15**). This single force is the total force represented by the distributed force and is located so as to create the same moment as the distributed force. For the uniformly distributed force in **Figure 6.15** we are able to find this location by inspection. In Chapter 8 we will show you how to find the location for non-uniformly distributed forces. The important point for your current work is that these distributed forces are included in the freebody diagram.

6.6 FLUID BOUNDARY CONDITIONS

The very nature of fluids acting on the boundary of a system is that they are distributed; for example, consider **Figure 6.16**. Like distributed loads associated with solid boundary conditions, the loads at fluid boundaries are included in a freebody diagram, either as distributed loads or as a single force and/or couple—see **Figure 6.16B**. In Chapter 8 we discuss in greater detail distributed loads due to fluids acting on the system.

6.7 FREEBODY DIAGRAM DETAILS

We now outline a process for drawing a freebody diagram of a system; this is the DRAW step in our Analysis Procedure.

1. Before diving into drawing, take time to **study the physical situation**. Consider what loads are present at boundaries and ask yourself if you have ever seen a similar connection. Study actual hardware (if available); pick it up or walk around it to really get a sense for how the loads act on the system. This inspection helps in making modeling assumptions (e.g., how loads interact with system, what is important, etc). **Classify the system as a planar or non-planar system**--- if the system can be

Figure 6.13

(A) (B)

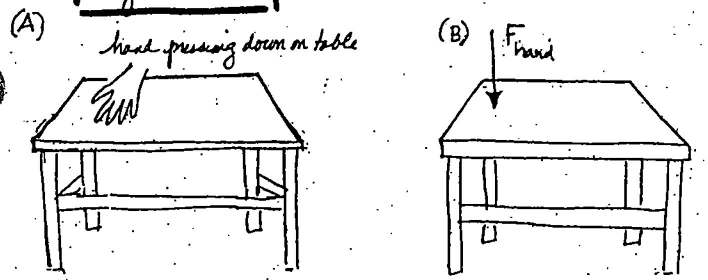
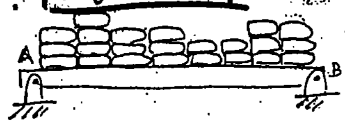
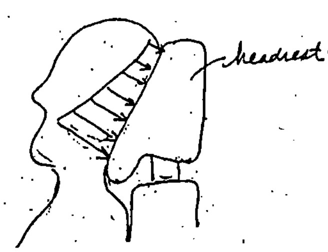


Figure 6.14



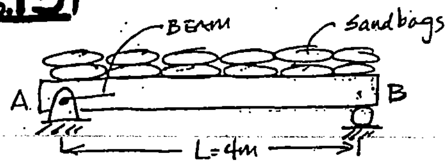
Situation: 60# bags of concrete are stacked on beam AB.



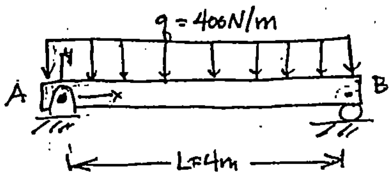
Situation: Pressure distribution between head and head restraint in rapidly decelerating automobile.

FIGURE 6.15

(A) Sandbags sitting on a beam



(B) Weight of sandbags represented as distributed load.



(C) Finding the equivalent force and its location.

To find the force that is equivalent to the distributed load:

$F_{equ} = q \cdot L = 400N/m \cdot 4m = 1600N$

To find location along length of beam of line of action of F_{equ} we find the location such that F_{equ} creates the same moment about any point as the distributed force. For this simple case we use inspection to say that F_{equ} must be located midspan of the beam (i.e., at x=2m). In Chapter 8 we introduce the formal math for finding this location.

(D) Representing the equivalent force

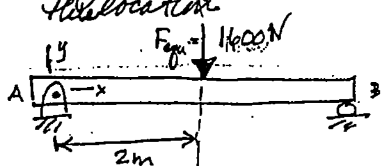
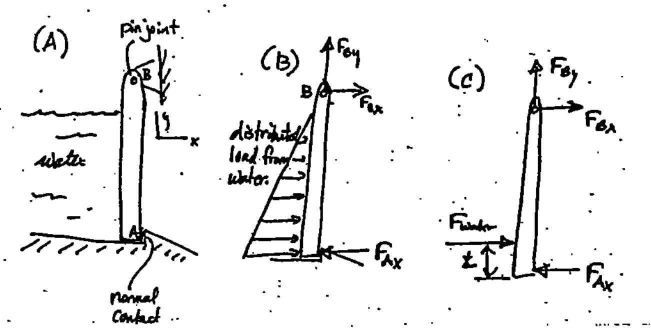


FIGURE 6.16



classified as planar, drawing the freebody diagram and writing and solving the conditions of equilibrium (as covered in the next chapter) all become easier. If you are unsure, consider the system to be non-planar. Also, consider asking for advice from others.

2. Define (either by imagining it or actually drawing it) a boundary that isolates the system from the rest of the world, then **draw the system** that is within the boundary. The drawing should contain enough detail so that distances and locations of loads acting on the system can be shown accurately. Sometimes multiple views of the system will be needed, especially if the system is non-planar. Establish an **overall coordinate system**. **State any assumptions** you make.

3. Identify **gravity forces** acting on the system and draw them at appropriate centers of gravity[1]. Include a variable label and the force magnitude (if known). Continue to make note of any assumptions you make.

4. Identify loads that are specified in the problem statement (so called **known loads**) that are due to either solid or fluid boundary conditions. Add these loads to the drawing of the system, placing each load at its point-of-application; identify each load with a variable label and magnitude. Also add known distributed loads to the drawing.

5. Identify **solid boundary conditions**, both those that act at discrete points and those that consist of distributed forces. If possible, classify each solid boundary condition as one of the standard connections (Table 6.1 for planar systems and Table 6.2 for non-planar systems) in order to identify the loads. If not, consider how the surroundings restrict motion (either translation and/or rotation) in order to identify the loads acting on the system at this boundary condition. Add these loads to the drawing of the system, placing each load at its point-of-application.

6. Identify **fluid boundary conditions**. Add these loads to the drawing of the system, showing them either as a distributed load or as its equivalent single-point load.

You now have a freebody diagram of a system, as well as a list of the assumptions made in creating it. A freebody diagram is an idealized model of a real system. By making

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[1] In Chapter 8 we will show how to find the center of gravity of a system.

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assumptions about the behavior of connections, dimensions, and the material, we are able to simplify the complexity of the real system into a model that we are able to analyze. We might want the model to exactly describe the real situation, but this is generally not an achievable goal, due to limitations of, for example, information, time, and/or money. We do want a model that we can trust and that gives results that closely approximate the real situation.

In creating a model, an engineer must decide which loads are significant. For example, a hinge is often modeled as having no friction about its axis. Yet for most hinges, grease, dust, and dirt have built up and there is actually some friction — some resistance to rotation. If friction is large enough to affect the behavior of the door (e.g., large enough to keep the door from swinging freely), we should include it in our model. But if the friction is small enough that the door can still swing freely, we may conclude that it is not significant for the problem at hand, and model the hinge loads as shown in **Figure 6.17**.

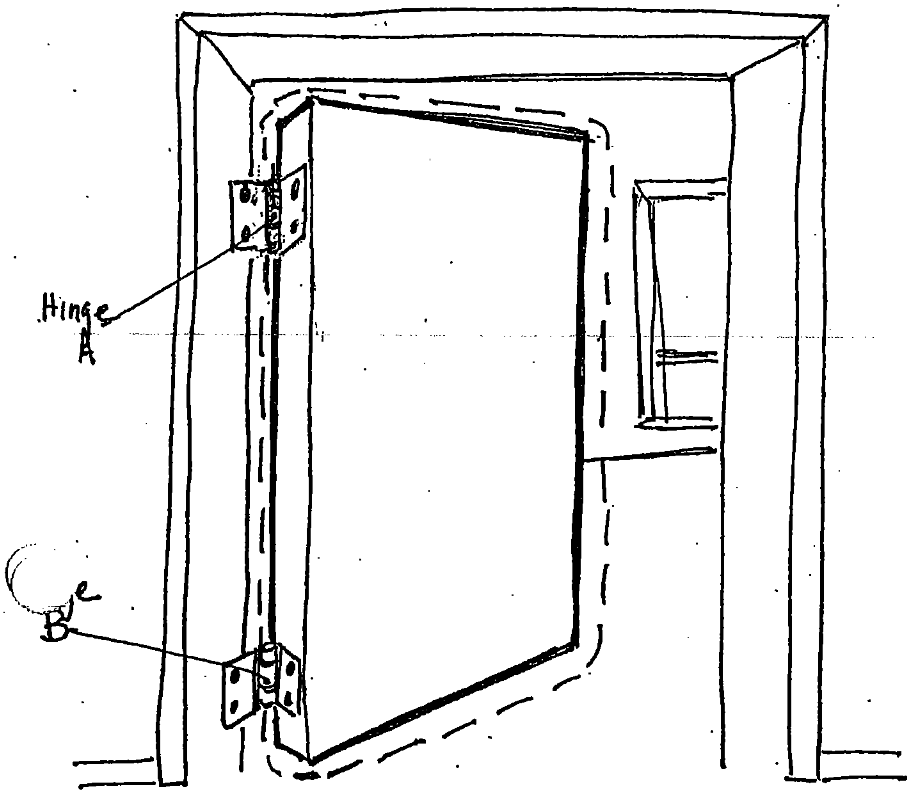
Often significance of loads is judged by the relative magnitude or location of the loads. For example, the weight of a sack of groceries is probably insignificant relative to the weight of an automobile that carries them home, but very significant if carried home by a cyclist on her bicycle. If you are in doubt about the significance of a load you should include it as significant. In many of the examples in this book, we will "set the stage" by making some of the assumptions regarding significance (that is, stating that a joint is frictionless). In others it will be up to you to judge the significance of a load based either on your own experience or on the advice of other engineers. Any loads that were considered to be too small to be significant are not included in the freebody diagram, but should be noted in the assumption list.

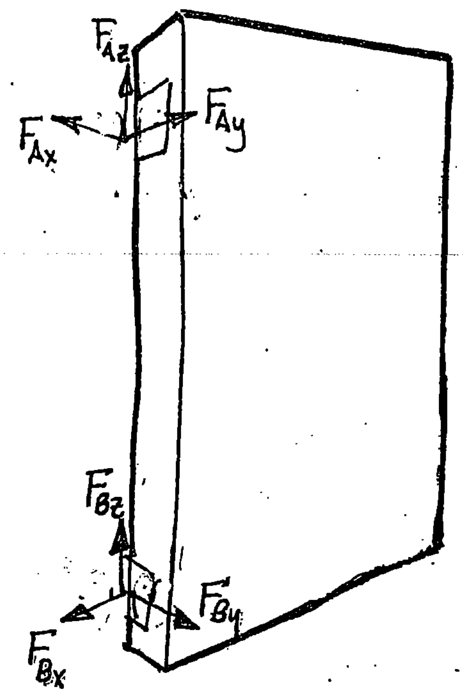
Sample Problem #6 Creating freebody diagrams (2D)
Sample Problem #7 Creating freebody diagrams (3D)

6.8 Keywords and Concepts

6.9 Homework Problems

Fall '01
Figure 6.17





6-18 B

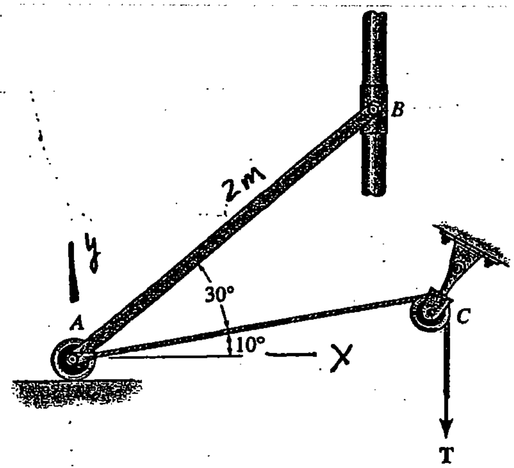
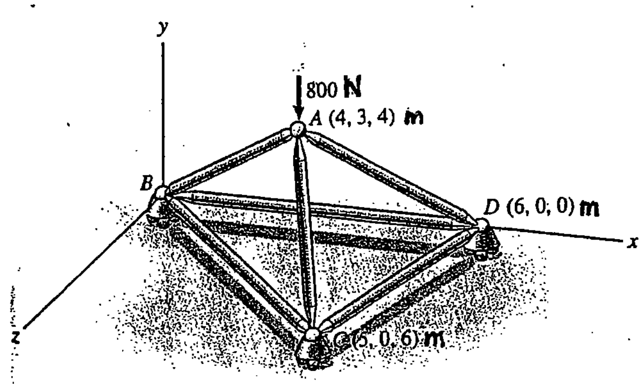
—CHAPTER 6—

DRAWING A FREEBODY DIAGRAM (SAMPLE PROBLEMS)

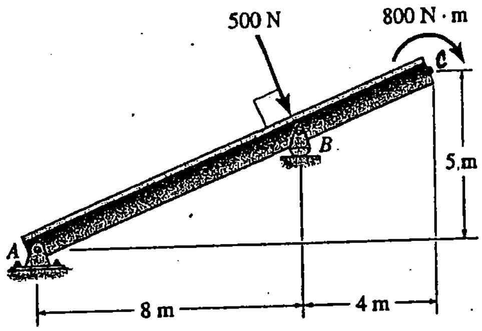
PAGE NO.

SP #1 Identifying planar and nonplanar systems

Task: Consider the description of each system and determine whether it can be classified as a planar or non-planar system.
(no systems are really planar, since we live in a 3D world. Even something as thin as paper has a third dimension. BUT, under certain conditions we can model/approximate/assume that it is planar).

Situation	Answer
A. The uniform arm AB weights 60 N. The system is taken as the arm and the wheel at A. Gravity acts in the y direction.  	The gravity force of 60 N is in the xy plane. Furthermore, points of application of boundary conditions at A (normal force). B (collar guide) and C (pulley) are also in the same plane. It is possible to define a single plane that contains all known forces and couples, gravity, and boundary condition application points---- therefore, this is a **planar system.**
B. The space truss has rollers at B, C, and D. It supports a vertical 800 N force at A. The system is taken as the space truss.  	It is not possible to define a single plane that contains the points of application of boundary conditions (B,C,D) and the 800 N force. Therefore, this is a **non-planar system.** (we did not consider gravity forces acting on the space frame---our answer would be unchanged even if we had).

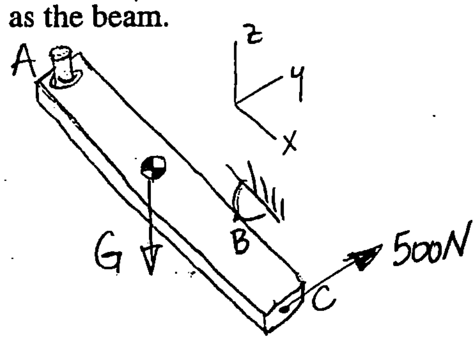
C. The beam AC is pinned to its surroundings at A and rests against a rocker at B. Ignore gravity. The system is taken as the beam.



The 800 Nm couple at C and the 500 N force are in the xy plane, as are the points of application of boundary conditions at A and B.

Therefore, this is a **planar system**.

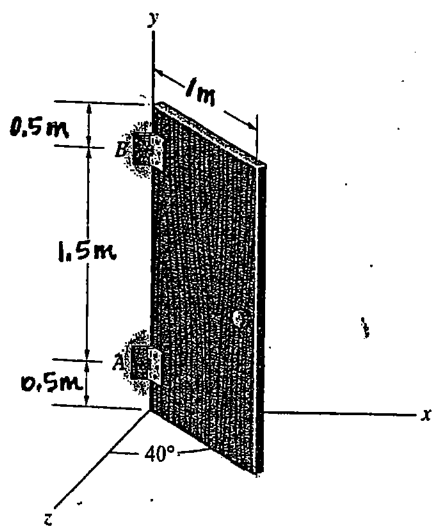
D. The beam AC is pinned to its surroundings at A and rests against a rocker at B. Gravity acts as shown. The system is taken as the beam.



It is not possible to define a single plane that contains the gravity force and the 500 N force.

Therefore, this is a **non-planar system**.

E. The 150 N door is supported at A and B by hinges. Gravity is in the y direction. The system is taken as the door.

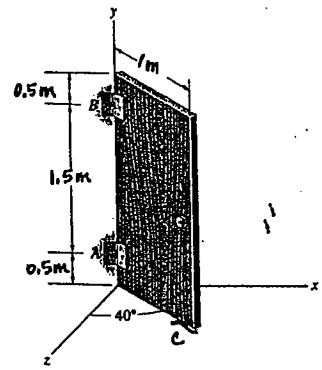


If we assume that the door is of uniform density, we place the 150 N at x=0.5m, y=1.25 m. Furthermore, if the door is thin relative to its other dimensions, this gravity force and the boundary conditions at A and B can be assumed to lie in the xy plane.

Therefore, this is a **planar system**.

<table>
<tr>
<td>

F. The 150 N door is supported at A and B by hinges. Someone attempts to open the door by applying a force of 30 N to the handle, but because of a high spot in the floor at C, is not able to. Gravity is in the y direction. The system is taken as the door.



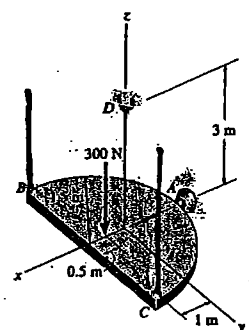
</td>
<td>

It is not possible to define a single plane that contains the gravity force, the 30 N force acting on the handle, and the boundary conditions at A, B and C.

Therefore, this is a **non-planar system**.

</td>
</tr>
<tr>
<td>

G. A semi-circular plate weights 300 N, which acts at center of gravity CG. Vertical cables support the plate at B and C, and a ball-and-socket joint supports the plate at D. The system is taken as the semi-circular plate.



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The xz plane is a plane of symmetry for this system---the portion of the system at +y (a quarter circle and cable force) is the mirror image of the portion of the system at –y (a quarter circle and cable force).

Therefore, it is possible to model this as a **planar system**.

</td>
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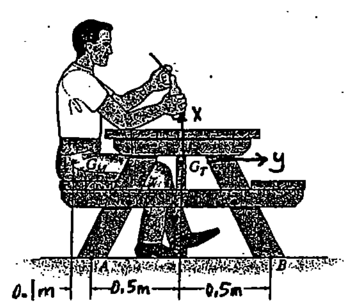
H. A man weighing 800 N sits at the picnic table. His center of gravity (G_m) is noted. The table weighs 200 N, that acts at G_t.. The system is taken as the picnic table.



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The xy plane is a plane of symmetry for this system---the portion of the system at +z (half of the picnic table and supports at A and B) is a mirror image of the portion of the system at –z (the other half of the picinic table supports at C and D).

Therefore, it is possible to model this as a **planar system**.

</td>
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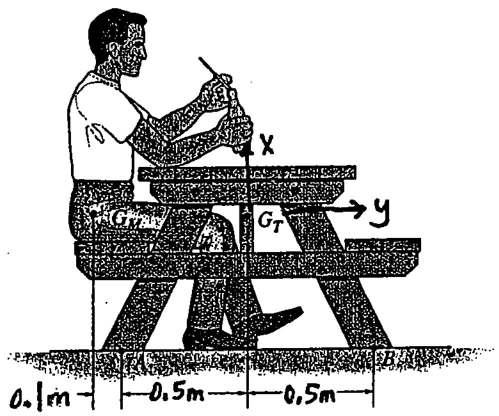
</table>

I. A child comes and sits down next to the man at the picnic table in H. The system is taken as the picnic table. 	With the child sitting next to the man, the xy plane is no longer a plane of symmetry. Therefore, this is a **non-planar system**

SP #2 Complete freebody diagrams

Task:
For each situation in Figures 1 and 3 the system has been defined as the block.
(a) Explain why these figures are not freebody diagrams.
(b) Create a freebody diagram of each system.

Figure 1

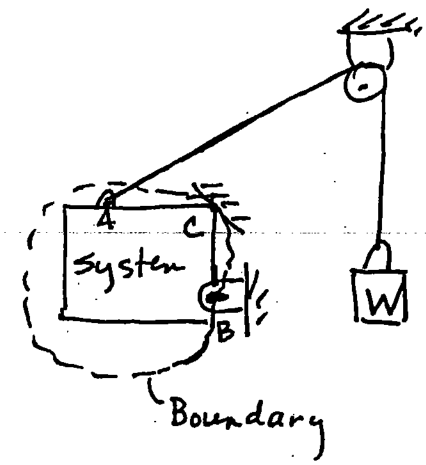
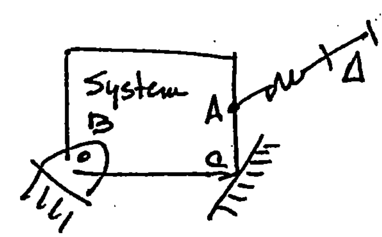


Figure 3



ANSWER:
(a) Both Figures 1 and 3 are not freebody diagrams because the system (the block) has not been isolated from its surroundings---at A and B it is still connected to the rest of the world. We note that the systems in both figures are planar as the boundary conditions all lie is a single plane.

Figure 1:

(b) We isolate the block using the boundary shown in **Figure 1**, establish an overall coordinate system (as shown), then we note that:

at B pin joint attaches the system to its surroundings. According to Table 6.1, a pin joint applies a force to the system. As we do not know the direction or magnitude of this force, we represent it as two components, $\mathbf{F_{Bx}}$ and $\mathbf{F_{By}}$. We have arbitrarily drawn each of these in the positive direction.

at C the system rests against a surface inclined at angle α relative to the horizontal. We are told that the surface is rough, so we must consider the presence of friction between the inclined surface and the system. According to Table 6.1, there will be a normal force $\mathbf{F_{Cnormal}}$ acting on the system, oriented perpendicular to the surface so as to push on the system, as shown. We do not know its magnitude. There is also the friction force $\mathbf{F_{Cfriction}}$ that is perpendicular to $\mathbf{F_{Cnormal}}$. We do not know the magnitude of $\mathbf{F_{Cfriction}}$, or whether it acts in the +y' or –y' direction (but have arbitrarily chosen to draw it in the +y' direction).

at A a cable pulls on the system, which we represent as a force of known direction ($\mathbf{F_A}$).

The block, with forces $\mathbf{F_{Bx}}$, $\mathbf{F_{By}}$, $\mathbf{F_{Cnormal}}$, $\mathbf{F_{Cfriction}}$ and $\mathbf{F_A}$, each drawn at respective point of application, constitutes a freebody diagram of the system.

Answer: Figure 2

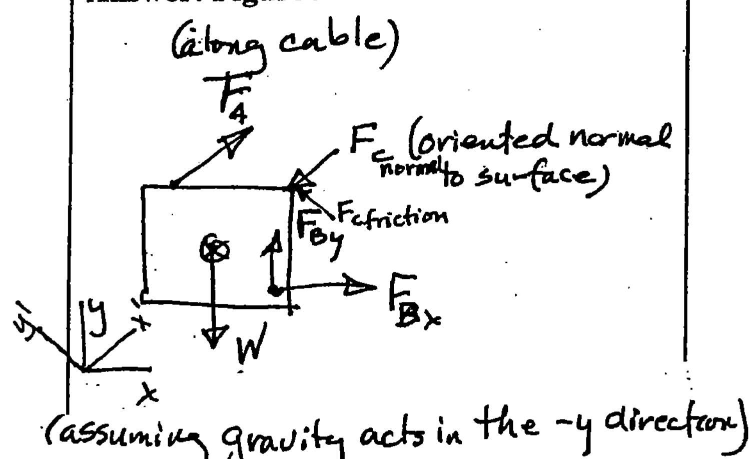


Figure 3:

(b) We isolate the block using the boundary shown shown in **Figure 3**, establish an overall coordinate system (as shown), then we note that:

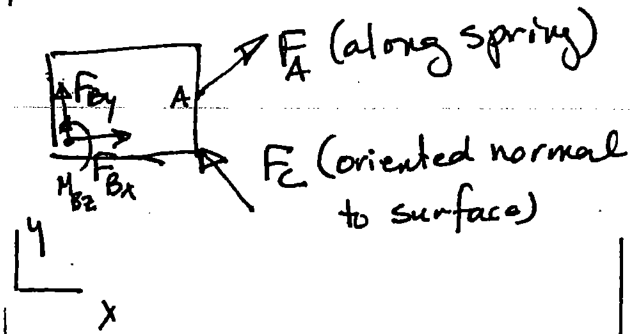
at B system is fixed to its surroundings. According to Table 6.1, a fixed boundary condition applies a force and a couple to the system. As we do not know the direction or magnitude of this force, we represent it as two components, $\mathbf{F_{Bx}}$ and $\mathbf{F_{By}}$. We know that the couple is about the z axis, but we do not know its magnitude; we represent it as $\mathbf{M_{Bz}}$. We have arbitrarily drawn $\mathbf{M_{Bz}}$ as a positive couple.

at C slot in the block rides against a pin. The pin pushes against the block in a direction perpendicular to the axis of the slot (Table 6.1), shown as $\mathbf{F_C}$.

at A spring pushes on the system since the spring compressed by an amount Δ. This force is along the spring axis and has magnitude of k(Δ), which we represent as a force of known direction ($\mathbf{F_A}$).

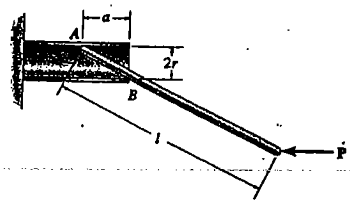
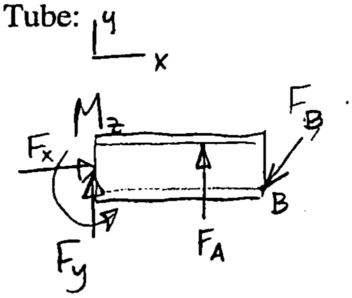
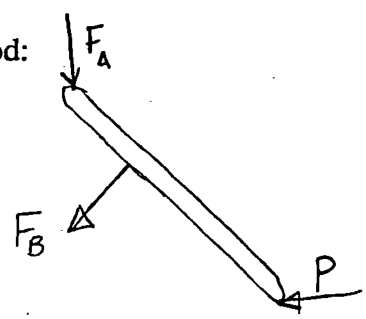
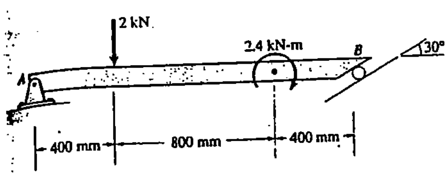
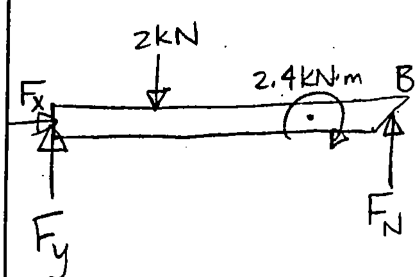
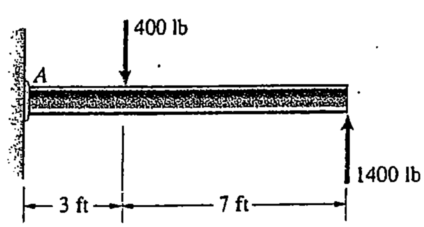
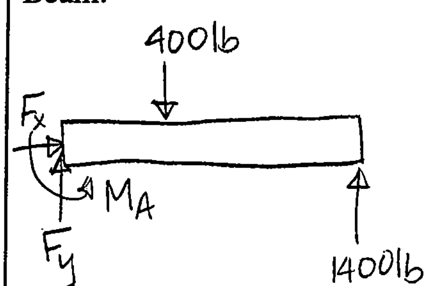
The block, with forces $\mathbf{F_{Bx}}$, $\mathbf{F_{By}}$, $\mathbf{F_C}$, and $\mathbf{F_A}$, and couple $\mathbf{M_{Bz}}$ each drawn at respective point of application, constitutes a freebody diagram of the system.

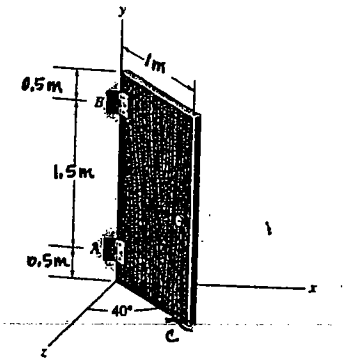
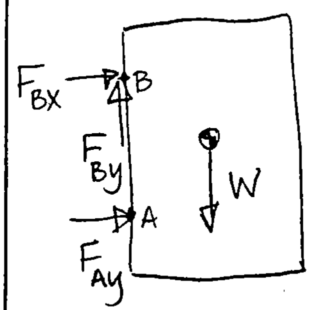
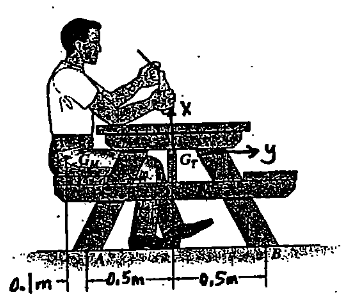
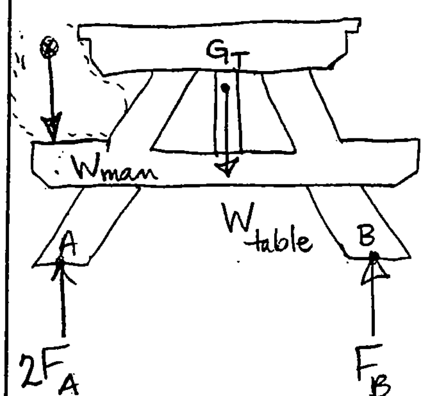
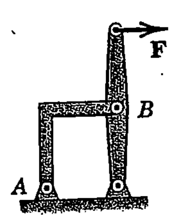
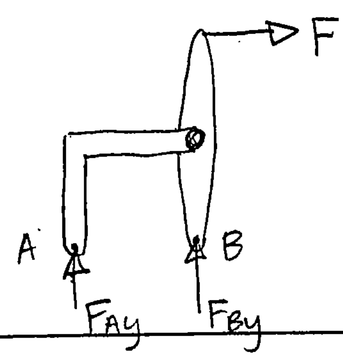
Answer: Figure 4

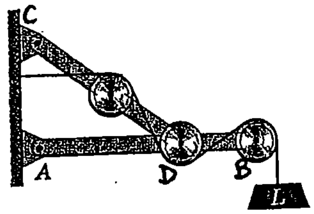
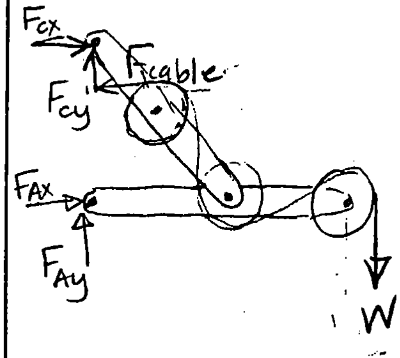
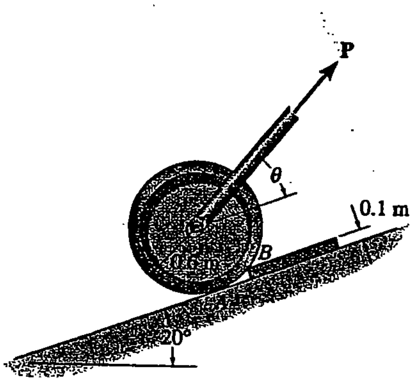
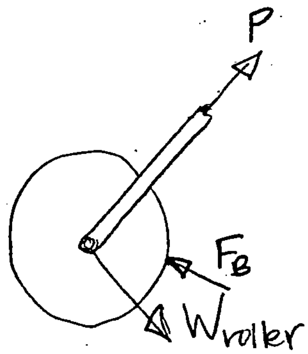


SP #3 Evaluating the correctness of freebody diagrams

Task: Consider the description of each planar system and determine whether the associated freebody diagram is wrong, incomplete or correct.

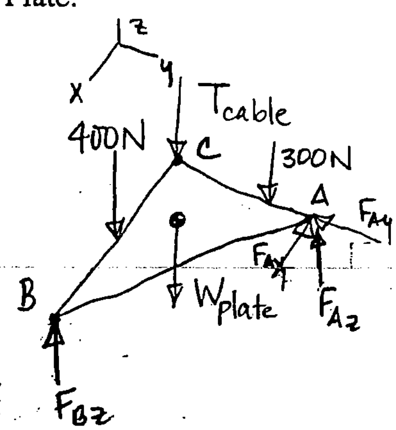
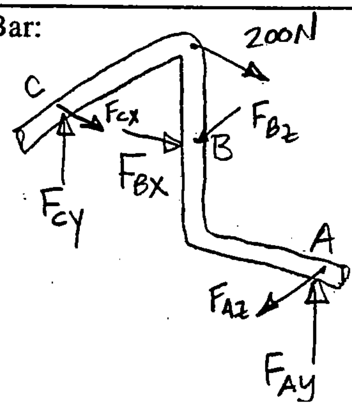
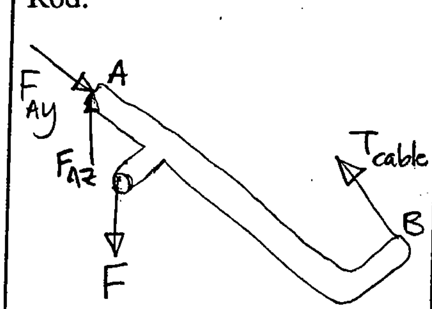
Situation	**Wrong, Incomplete or Correct Freebody Diagram?**	**Answer:**
A. The thin rod is supported by the smooth tube. The tube is fixed to the wall. 	Tube:  Rod: 	Tube: this freebody diagram is correct. It accounts for the fixed end of the left, and the normal contact between the tube and the rod. Notice that the couple at the fixed boundary condition could be shown as two forces, and not as a pure moment (which is just fine). Rod: this freebody diagram is not correct. The force $\mathbf{F_B}$ in this diagram (which represents the normal force of the tube pushing on the rod) should be in the other direction. As shown, the tube is pulling on the rod.
B. A beam is pinned at A and rests against a smooth incline at B. 	Beam: 	This freebody diagram is not correct. The normal force acting on the beam at B should be oriented perpendicular to the inclined surface.
C. A beam is fixed at A. 	Beam: 	This freebody diagram is correct

D. A door that weighs W hangs from hinges at A and B. The hinge at A is able to apply a horizontal force to the door. The hinge at B is able to apply both horizontal and vertical forces to the door. 	Door: 	This freebody diagram is <u>correct</u>
E. A man weighing 800 N sits at the picnic table. His center of gravity (G_m) is noted. The table weighs 200 N, that acts at G_t. The system is taken as the picnic table. 	Table: 	This freebody diagram is <u>not correct</u>. The label for the normal force acting on the table at B should be $2F_B$, (not F_B), as the force vector at B represents the normal force for two legs.
F. Supporting angle bracket for a frame, with pin joints at each end. 	Angle Bracket: 	This freebody diagram is <u>incomplete</u>. At each pin joint a force in the x and y directions should be shown.

G. A frame consisting of members AB and CD supports the pulleys, cable and weight.  	Whole frame:  	This freebody diagram is correct. It includes the forces at pins A and B, the cable tension $\mathbf{F_{cable}}$ pulling on the frame, and the gravity force from block L.
H. A 50-kg roller rests against a smooth step  	Roller:  	This freebody diagram is not correct. The gravity force should be drawn in the y direction.

SP #4 Evaluating the correctness of freebody diagrams

Task: Consider the description of each nonplanar system and determine whether the associated freebody diagram is wrong, incomplete or correct.

Situation	**Wrong, Incomplete or Correct Freebody Diagram?**	**Answer:**
A. The triangular plate ABC is supported by a ball-and-socket at A, a roller at B and a cable at C. The plate weighs 100 N.	Plate: 	This freebody diagram is not correct. Because the cable is in tension, it will pull on the plate in the +z direction (not in the push on it in the –z direciton, as shown).
B. A bar is supported by journal bearings at A, B and C.	Bar: 	This freebody diagram is correct. Because there is not a single journal bearing supporting the system, each bearing applies force (and not couple) to the system.
C. A rod is supported at A by a journal bearing and a cable at that extends from B to A.	Rod: 	This freebody diagram is not complete. Because there is a single journal bearing supporting the system, the bearing at A also applies pure moments abou the y and z axes to the rod.