—CHAPTER 5 —

MOMENTS

5.1 PROPERTIES AND CHARACTERISTICS OF A MOMENT

5.2 MATHEMATICAL REPRESENTATION OF A MOMENT

5.3 ANOTHER MATHEMATICAL APPROACH TO FINDING THE MOMENT

5.4 EQUIVALENT LOADS

5.5 REVISITING GENERAL STEPS FOR ANALYSIS

5.6 CHAPTER SUMMARY AND KEY WORDS

5.7 HOMEWORK PROBLEMS

In the previous chapter we considered the forces that push and pull on systems. We described the various types of forces (gravitational, normal contact, friction, fluid contact, tension, compression, shear), outlined methods of representing and manipulating forces mathematically and graphically, and showed how to create freebody diagrams that illustrate the relationship between forces and systems.

In this chapter we consider how forces not only push and pull, but also tend to twist, tip, turn, and rock the systems on which they act. This tendency is illustrated with a can of soda pop (**Figure 5.1**). If you place your hand near the bottom of the can and push, the can slides across the table. If you place your hand near the top of the can and push, the can tips over. This chapter is about the tipping tendency of force, which is called moment*.

Some designs function because of moments, including see-saws, balance scales, can openers, and torsion-bar suspensions. Other designs must be sized to withstand moments such as skyscrapers, diving boards and airplane wings. *(sketches of devices)*

We begin this chapter by presenting the properties and characteristics of moments. Two formal mathematical methods for calculating moments are outlined and then situations involving multiple forces are addressed. Finally, how moments fit into the general Analysis Procedure presented in Chapter 1 are discussed.

By the end of this chapter, you will be able to:

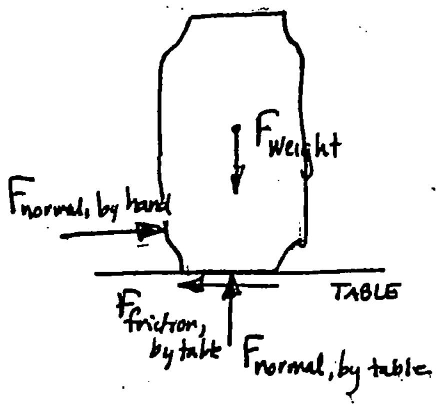
- apply the concept of moment to finding the moment created by a force;
- represent a moment numerically and graphically;
- find the equivalent moment and equivalent force due to a number of forces acting on a system;
- apply the concepts of moment and force in static analysis.

---

* Some physics textbooks use the word **torque** for what we are calling moment---namely, the tendency of a force to cause tipping or rotation. Engineers generally use the word torque to describe a moment created in conjunction with a machine (a special type of structure). Machines and torques are discussed in Chapter 9.

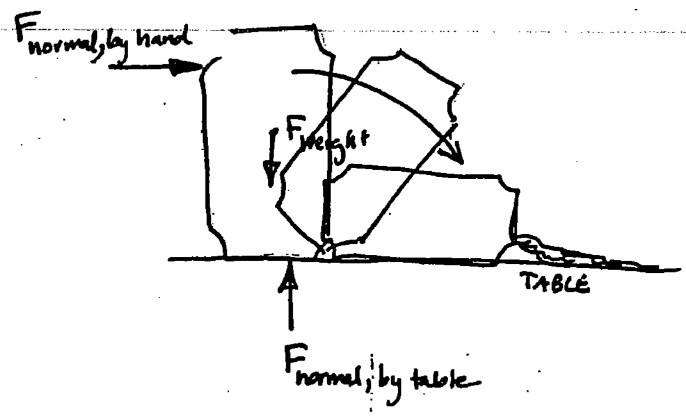
FIG 5.1

(a)



F normal, by hand
F weight
TABLE
F friction, by table
F normal, by table

(b)



F normal, by hand
F weight
TABLE
F normal, by table

5.1 PROPERTIES AND CHARACTERISTICS OF A MOMENT

Imagine you have just changed a flat tire on your car and are replacing the lug nuts that bolt the wheel to the hub. You use a wrench to tighten each nut onto a bolt, as in **Figure 5.2**. By pushing downward on the wrench, you are applying a **moment** to the nut. This moment is the tendency of the force to twist the nut about the axis of the bolt (which in the figure is aligned with the z axis). The size (or magnitude) of the moment is the product of the distance from your hand to the z axis and the magnitude of the force.

A moment is created by a force that is offset relative to the origin of a set of reference axes. It is a vector quantity and so has both magnitude and direction. For example, the moment created when tightening the lug nut has magnitude M that is the product of the distance D from push force to the z axis and the magnitude of the push force F (i.e., $M = D \cdot F$). It is specified in newton-meters (N-m) in SI units and inch-pounds (in-lbs) in U.S. Customary units. The moment's direction, as we'll see below, is along the negative z axis when you face the wheel, as depicted in **Figure 5.2.**

In working with moments we must concern ourselves with reference axes, the force's position relative to the reference axes, magnitude of the moment, direction and sense of the moment, and graphical depiction of the moment, as discussed below:

<u>Reference axes</u>— The moment created by a force depends on the position of the force relative to a set of right handed reference axes. Engineers think carefully about how to orient the reference axes, generally orienting them parallel to an overall coordinate system. Often the origin of the reference axes will be located at a boundary of the freebody diagram or at some other point of particular interest.

<u>Position vector</u>—A position vector is any vector that runs from the origin of the reference axes to the line of action of the force (**Figure 5.3**). Because it is a vector, it has magnitude (length) and direction (from the origin to the line of action), and can be manipulated with vector operations. Position vectors are commonly specified in terms of their rectangular components (e.g., position vector **r** is specified as $r_x\mathbf{i}+r_y\mathbf{j}+r_z\mathbf{k}$).

The most straightforward position vector for the wrench force **F** relative to the reference axes is illustrated in **Figure 5.4A** as a vector that runs from the origin to the point of application of **F**; this vector is defined as $\mathbf{r}_1=250\ \mathbf{i}$ (mm). It is just one of a family of position vectors as *any* vector from the origin to the line of action of the force is a position

FIG 5.2

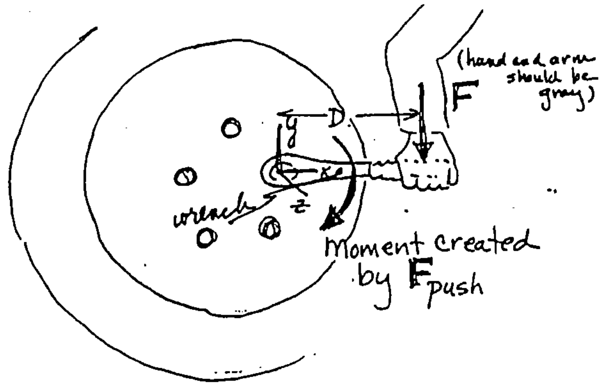


FIG 5.3

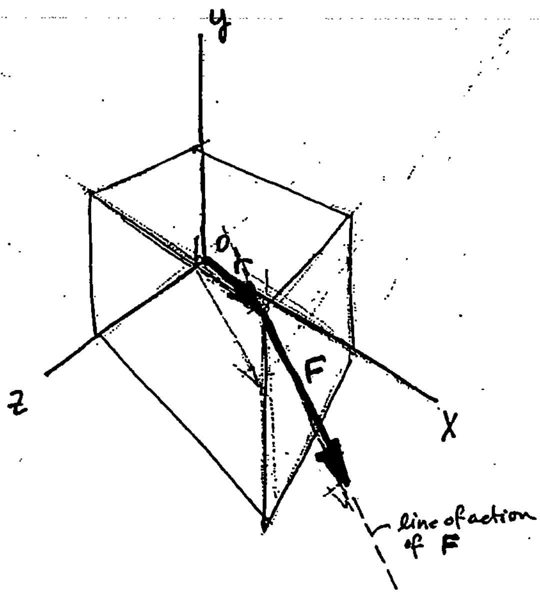
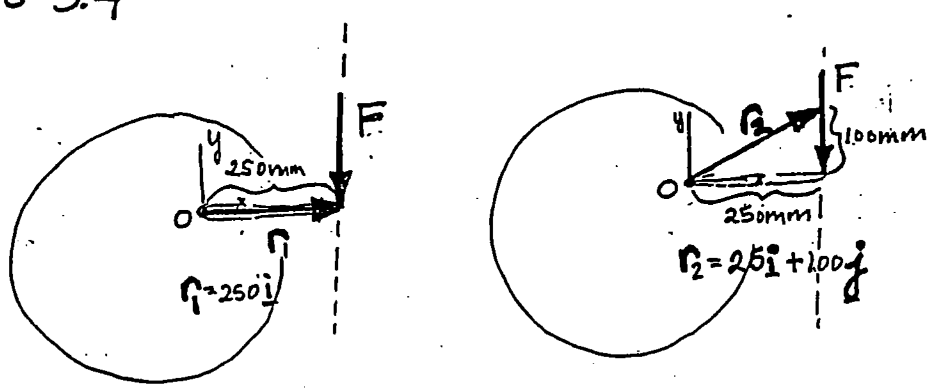


FIG 5.4



(A) (B)

vector. For example, another position vector for **F** is $\mathbf{r}_2$=250 **i** + 100 **j** (mm) (**Figure 5.4B**).

Now consider the situation depicted in **Figure 5.5A**; an individual pulls with a force of 400 N on a rope tied to a branch (he is attempting to pull down the nearly sawn-through branch). We can represent this force as shown in **Figure 5.5B**. A position vector $\mathbf{r}_1$= {2 **i** + 6 **j** - 9 **k** }m is illustrated and is just one of a family of position vectors. For example, another position vector is $\mathbf{r}_2$= {0.5**i** +1.5 **j** + 1.0 **k** }m (**Figure 5.5C**).

When choosing a position vector keep in mind that it need not lie along a physical connection between the origin and the line of action of the force. Generally, the position vector you should choose is the one that is easiest to define with the information known about the physical system. Examples of position vectors in two physical systems are shown in **Figure 5.6**; in each case, the position vector that is easiest to define with available dimensions is highlighted.

Synonyms for position vector commonly used in engineering practice are "moment arm vector," "moment arm," "lever arm," or "offset."

<u>Magnitude of moment</u>— The magnitude M of a moment is the product of the magnitudes of the position vector (r) and the force component perpendicular to the position vector. This force component is ($F\sin\theta$), where θ is the angle between the position vector and the line of action of the force, as illustrated in **Figure 5.7**. Therefore, we can write the magnitude of the moment as:

$$M = r\,(F \sin\theta) \qquad (5.1)$$

where

r = magnitude of position vector

(for a position vector expressed in rectangular components $r = (r_x^2+r_y^2+r_z^2)^{0.5}$),

F = magnitude of force

(for a force expressed in rectangular components $F = (F_x^2+F_y^2+F_z^2)^{0.5}$), and

θ = angle between position vector and force when these two vectors are placed tail-to-tail and is such that $0° < \theta < 180°$.

FIG 5.5

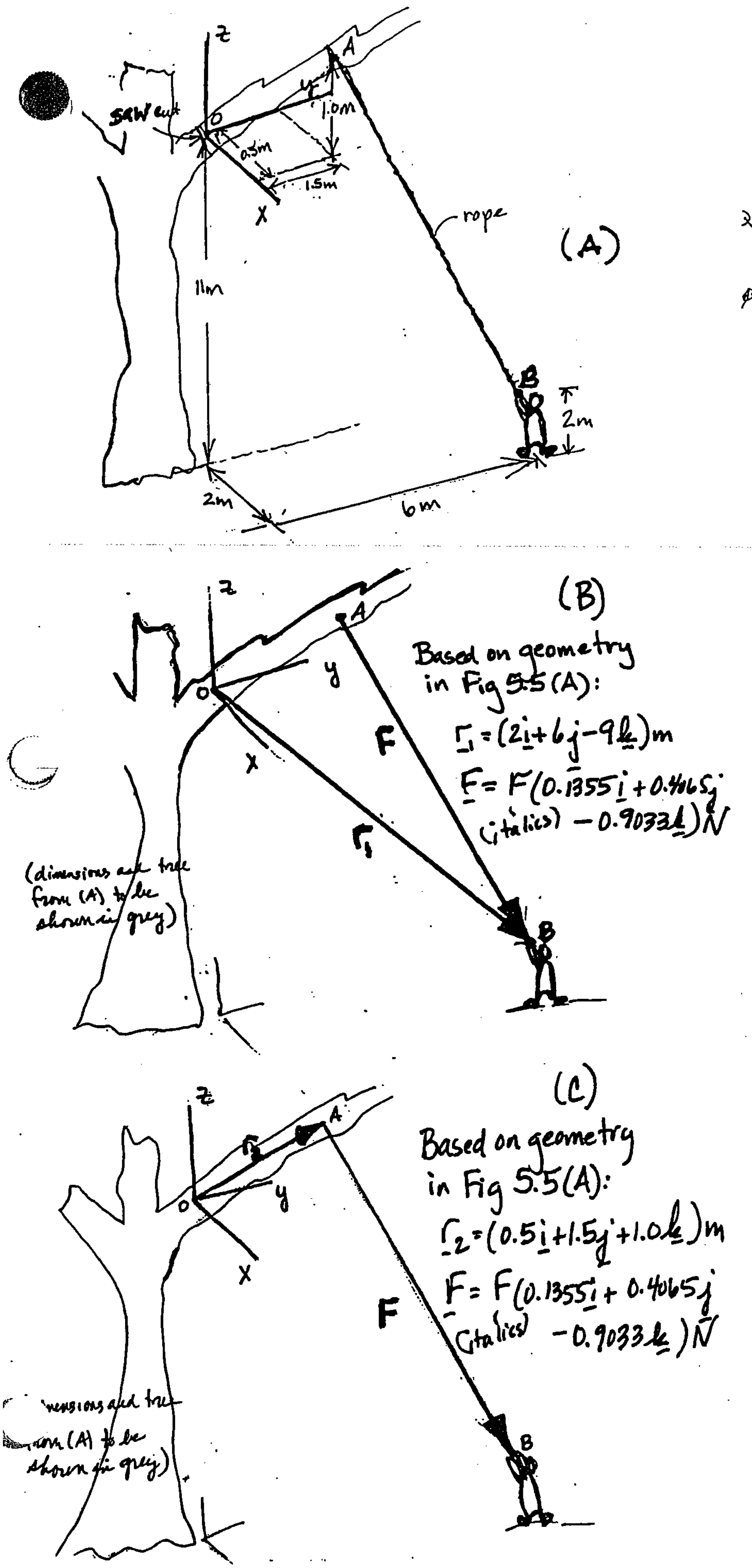


FIG 5.6

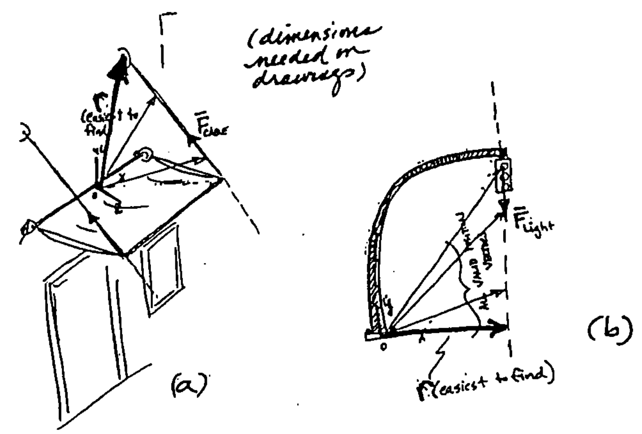
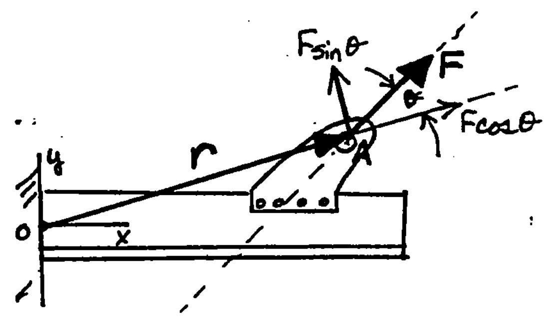


FIG 5.7



Then use Equation 5.1 to determine the magnitude of the moment for the lug nut example with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ is illustrated in **Figure 5.8**. With $\mathbf{r}_1$, the angle θ is 90°—therefore the component of **F** that is perpendicular to $\mathbf{r}_1$ is simply *F*. With $\mathbf{r}_2$, the angle θ is 111.8°—therefore the component of **F** that is perpendicular to $\mathbf{r}_2$ is ($F \sin 111.8°$). *Remember, only the component of F perpendicular to r (F sin θ) creates the moment. The component of F along r (F cos θ) does not create a moment.*

We end up with the same value *M*, no matter which position vector we used. This leads us to the conclusion that *the magnitude of the moment created by a force relative to the origin of a set of reference axes is independent of the position vector.*

The use of Equation 5.1 to find the magnitude of the moment created by rope tension pulling on the branch in **Figure 5.5** is illustrated in **Figure 5.9**.

Sense and direction of moment—To tighten the lug nut onto the bolt, a force is applied at the end of a wrench so that the resulting moment twists the nut about the z axis in the clockwise direction, as viewed in front of the wheel. To loosen the nut, we would apply the force so that the resulting moment twists the nut in the counterclockwise direction. The terms "clockwise" and "counterclockwise" refer to the **sense** of the rotation about a particular axis. They are defined relative to standing on the positive axis of rotation (in this case, the positive z axis) and looking back at the origin, as illustrated in **Figure 5.10**. The axis of rotation is the **direction** of the moment, as detailed below.

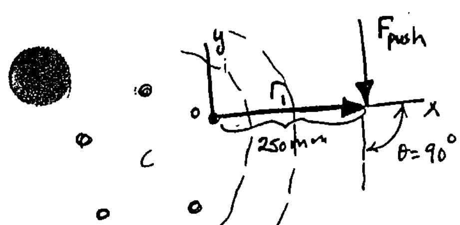
The **right hand rule** enables us to formalize a procedure for determining the sense and direction of a moment. Consider a force applied to the wrench that results in a counterclockwise (*loosening*) rotation (**Figure 5.11A**). To determine the moment's sense align the fingers of your right hand with the position vector (**Figure 5.11B**). Then rotate your palm so that you can curl your fingers toward the force (**Figure 5.11C**). The "curl" (counterclockwise in this particular example) defines the sense of the moment. The direction of your thumb defines the direction of the moment (along the positive z axis in this particular example). With the positive z axis aligned with your thumb and your fingers curled counterclockwise we say that the moment is in the **positive k** (+**k**) direction.

When you follow the right hand rule for the case where the wrench moves clockwise so as to tighten the lug nut, as in **Figure 5.12A**, you find you have to rotate your hand such that your thumb tip rotates into the page, forcing your thumb to align with the negative z

FIG 5.8

(a)

(wheel/tire shown in gray)



$M = r_1 F_{push} \sin \theta$ (θ = 90°)

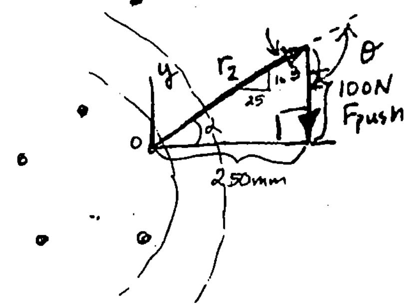
Therefore:

$M = (250) F_{push}$ N

$M = (250 F_{push})$ Nmm

The same!

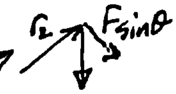
(b)



$\alpha = \tan^{-1}\left(\frac{100}{250}\right) = 21.8°$

$\beta = 90° - 21.8° = 68.2°$

$\theta = 180° - \beta = 111.8°$



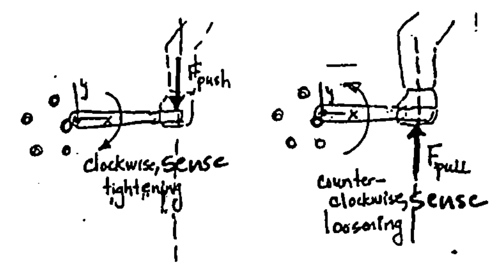
$M = r_2 (F_{push} \sin \theta)$ ($r_2 = \sqrt{100^2 + 250^2}$, θ = 111.8°)

Therefore:

$M = (269 \text{ mm}) F_{push} \sin 111.8°$ ($\sin 111.8° = 0.928$)

$M = (250 F_{push})$ Nmm

FIG 5.10



(A) (B)

FIG 5.11

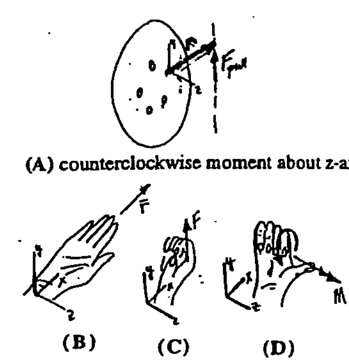
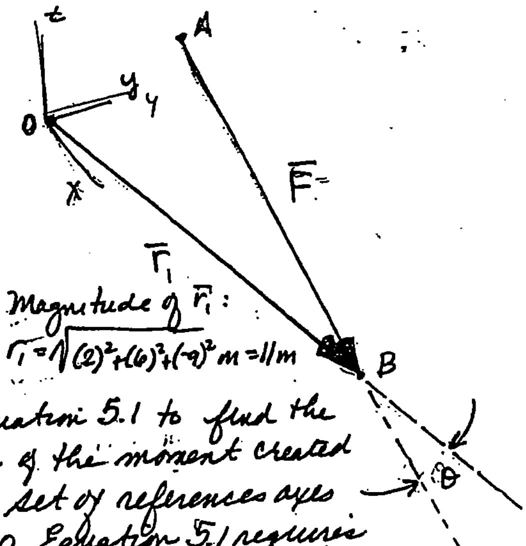


FIG 5.9



Magnitude of r_1:

$r_1 = \sqrt{(2)^2 + (6)^2 + (-9)^2}\ m = 11 m$

- Will use Equation 5.1 to find the magnitude of the moment created by F about set of references axes located at O. Equation 5.1 requires that we know the angle θ between r_1 and F. We can find this angle using Equation 4.27:

Equation 4.27: $\theta = \cos^{-1}\left[\frac{V_{1x}V_{2x} + V_{1y}V_{2y} + V_{1z}V_{2z}}{V_1 V_2}\right]$ (italic)

For r_1 and F this equation is:

$\theta = \cos^{-1}\left[\frac{F(0.1355)2 + F(0.4065) + F(-0.9033)(-9)}{F(11)}\right]$ (italic)

θ = 9.79°

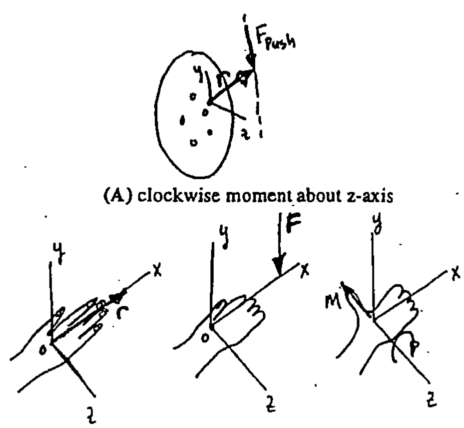
Now use Equation 5.1 to find the moment's magnitude

$M = r_1 F \sin \theta$ (italic)

$= (11 m)(F\ N) \sin 9.79°$

M = 1.87 F Nm

FIG 5.12



axis; in addition, your fingers are curled clockwise (**Figure 5.12B**). With the negative z axis aligned with your thumb and your fingers curled clockwise, we say that the moment is in the **negative k** (-**k**) direction.

The right hand rule also works when considering the sense and direction of moments about the x and y axes. For example, a direction of **positive i** (+**i**) or **negative i** (-**i**), refer to counterclockwise or clockwise rotation, respectively, about the x axis, as illustrated in **Figure 5.13A.** Similarly, **positive j** (+**j**) or **negative j** (-**j**) refer to counterclockwise or clockwise rotation, respectively, about the y axis, as illustrated in **Figure 5.13B.**

The position vector and force define a plane (the curl of your fingers), and the moment is about an axis perpendicular to this plane (the direction of your thumb). Therefore, the direction of the moment is perpendicular to both the direction of the position vector and the direction of the force vector. An important implication of this is *that a force cannot create a moment about an axis that is parallel to the force (because if the force and axis are parallel, it is impossible for them to be perpendicular to one another!).* This means that a force in the x direction cannot create a moment about the x axis, a force in the y direction cannot create a moment about the y axis, and a force in the z direction cannot create a moment about the z axis.

<u>Graphical representation of a moment</u>—We have been representing moments in drawings (e.g., **Figure 5.13**) with an arrow-headed arc. The arrow shows the sense of the moment. The magnitude of the moment (if known) is written next to the arc, as illustrated in **Figure 5.14A.**

A moment can also be represented on drawings with a double-headed arrow (**Figure 5.14B**). The direction of the arrow represents the direction of the moment (positive direction if it points along the positive axis and negative direction if it points along the negative axis). The sense of the moment is about the line along which the arrow lies (the sense is counterclockwise if the arrow points in the positive direction and clockwise if it points in the negative direction). If the magnitude of the moment is known, it is written next to the arrow and/or the length of the arrow is drawn in proportion to the magnitude.

Still another graphical representation of a moment is a single-headed arrow with an arc (**Figure 5.14C**). These various representations of moments in **Figure 5.14** enable

FIG 5.13

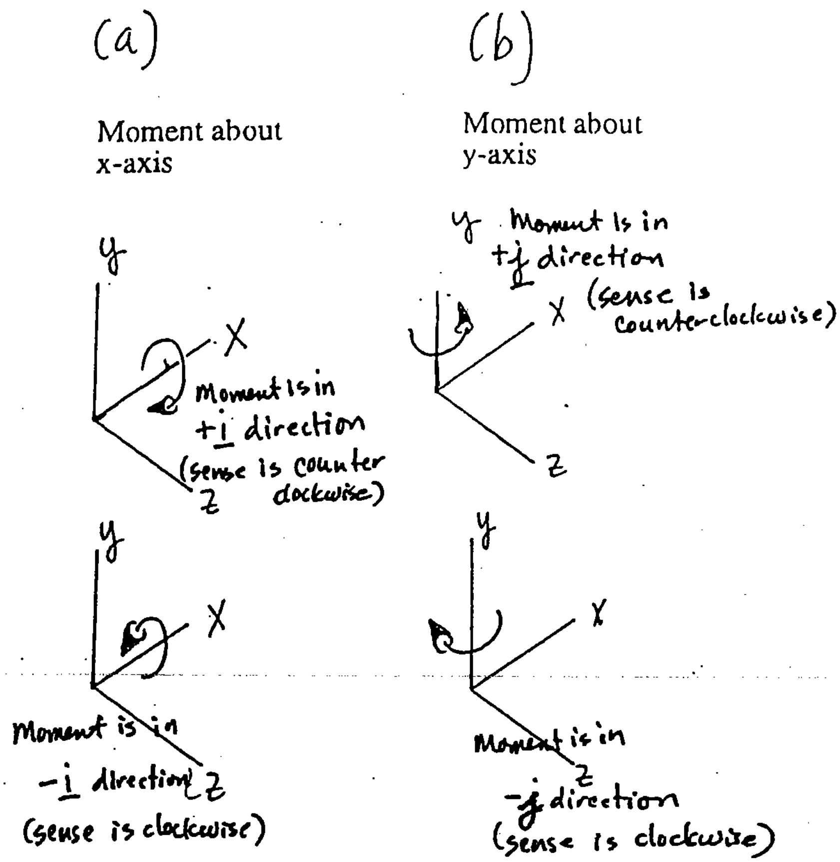
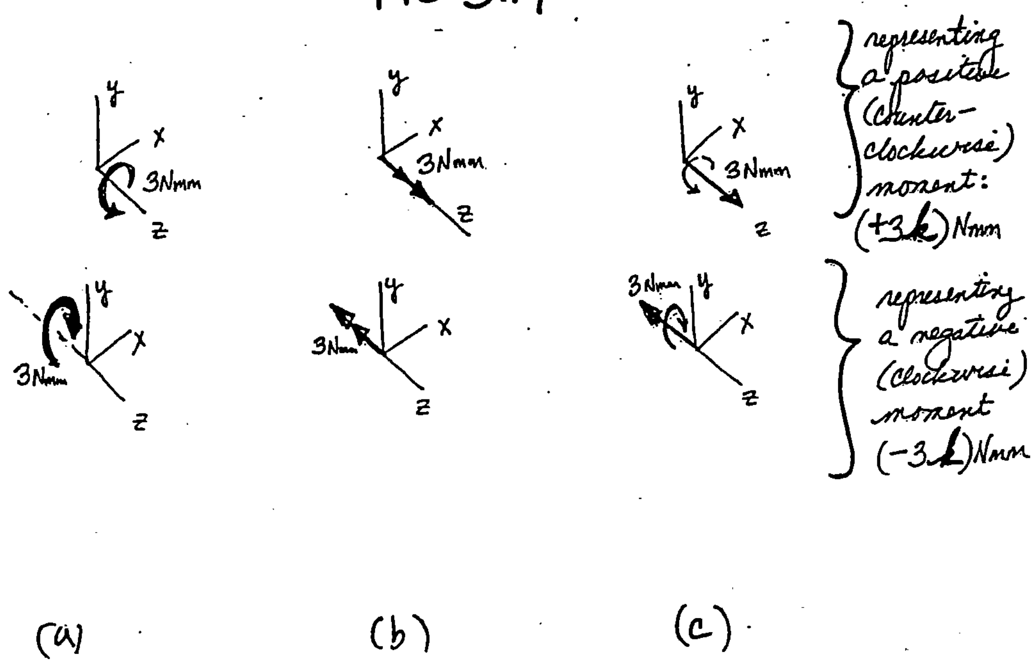


FIG 5.14



moments to be readily differentiated from forces (which are depicted as single-headed, straight arrows).

Sample Problem #1 Specifying the Position Vector
Sample Problem #2 The magnitude of the moment
Sample Problem #3 Increasing the magnitude of the moment

5.2 MATHEMATICAL REPRESENTATION OF A MOMENT

As discussed in Section 5.1, a moment **M** is a vector quantity, which means it has both magnitude and direction. It is frequently convenient to represent a moment in terms of its scalar components, i.e.,:

$$\mathbf{M}=M_x\mathbf{i}+M_y\mathbf{j}+M_z\mathbf{k} \qquad (5.2)$$

We can find the magnitude of the moment *M* with the scalar components as:

$$M=(M_x^2+M_y^2+M_z^2)^{0.5} \qquad (5.3)$$

(Figure 5.15A).

The axis about which the moment acts is defined in terms of the direction cosines of the axis:

$$\cos\theta_x= M_x /M$$
$$\cos\theta_y= M_y /M \qquad (5.4)$$
$$\cos\theta_z= M_z /M$$

(Figure 5.15B). The axis can also be described by a unit vector **n**:

$$\mathbf{n}= \cos\theta_x\,\mathbf{i}+ \cos\theta_y\,\mathbf{j}+ \cos\theta_z\,\mathbf{k} \qquad (5.5)$$

The idea of representing the direction of a vector (in this case, **M**) in terms of its direction cosines or a unit vector along its line of action should sound familiar. In Chapter 4 we represented force **F** in the same manner (mention here the equations from Chapter 4, and Table 4.2).

****