—CHAPTER 4—

FORCES

FORCES

All of us experience forces—all the pushes, pulls, tugs, and shoves of everyday life. Some of these forces are welcome—the up-down-sideways jostles of a roller coaster ride, and some are unwelcome—the thud and bump when two cars roll into each other. Forces operate can openers, automobiles, ski lifts and airplanes, and are what buildings, bridges, and ships must stand up to.

Engineers must consider how forces affect the structures, buildings, devices, and machines they design, manufacture and maintain. For example, a civil engineer* designing a dam would think about the water pushing against the dam and would ask "will the steel tie-downs to the bedrock be strong enough?" A mechanical engineer† designing the landing gear for an airplane would think about the forces applied during landing and ask "will the size of the landing gear forging be sufficient to prevent failure after repeated landings?"

You must consider forces if you propped a ladder against a building or tree to wash a window or rescue a kitten; if the angle of the ladder relative to the ground is too steep the ladder will tend to tip; if not steep enough its foot will tend to slide. The question that you might ask is, "will I be able to get just the right position for the ladder so that I can safely accomplish my task?"

This chapter looks at forces—what they are, how to categorize them, and how to represent them. Learning how to identify and represent forces is the first step in developing the thinking and analysis skills that will enable you to evaluate systems.

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*Civil engineers are responsible for planning, designing, and constructing the infrastructure of our civilization. This includes buildings, bridges, power plants, transportation systems, water supply and treatments systems, and much more. The civil engineer is called upon to apply physical (and, in some cases, chemical and biological) principles, assess social and environmental impact, and evaluate the costs and benefits of infrastructure projects.

† Mechanical engineers work in a variety of industries, from transportation to product manufacturing to energy generation to consumer products to applied research. This work involves the design, manufacture, and maintenance of products or systems to meet human needs. The mechanical engineer is called upon to use knowledge of physical principles and their application, an understanding of existing products, and an imagination of what products might be.

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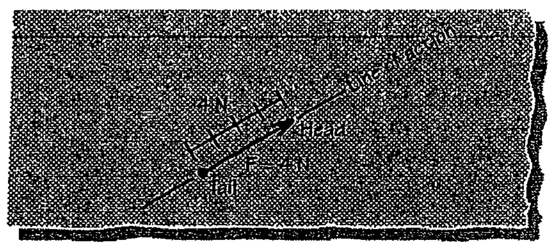
4.1 WHAT ARE FORCES?

The term **force** is used to describe any interaction between a system and the rest of the world that tends to affect the state of motion of the system. The magnitude of the force is related to the magnitude of the effect. Forces cannot be seen and in a sense don't exist—but are a rather useful concept.

Forces are specified in newtons (N) in SI units and in pounds (lbs) in U.S. Customary units. The conversion between the two systems is 4.4482 newtons = 1 pound, 1 newton= 0.2248 pound. Forces range from very small (e.g., 0.000 000 5 N gravitational pull exerted by Mars on an earth-bound engineering student) to very large (e.g., 100 tons = 889 660 N weight of a Caterpillar™ D11N tractor). In addition, forces are vector quantities; this means that they have both **magnitude** (size) and **direction** associated with them. Graphically, a force is represented by an arrow with a head and a tail (**Figure 4.1**). The direction from the tail of the arrow to its head represents the direction of the force, and the length of the arrow is commonly drawn proportional to the magnitude of the force. The magnitude of the force (if known) is written next to the arrow. The line along which the force acts is called the **line of action** of the force.

Physicists have traditionally identified four basic forces: gravitational, electromagnetic, weak, and strong. The relative strengths of these forces are strong 1, electromagnetic 10^{-2}, weak 10^{-7}, and gravitational 10^{-38}. Generally, **gravitational forces** are of concern to engineers considering equilibrium of systems. Also of concern are electromagnetic forces that result from the interaction of electrical and magnetic fields at the atomic and subatomic levels—we will refer to these as **contact and bonding forces.** The strong force (which keeps every atomic nucleus intact) and the weak force (which is a factor only in radioactive decay) are significant only at the subatomic level and will not be considered further here.

FIG 4.1



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4.2 Gravitational Forces

The force of gravity is always present. It is the mutual attraction between any two bodies anywhere in the universe. The magnitude of the gravitational force between any two bodies is directly proportional to the product of their masses and inversely proportional to square of the distance between them. The **universal gravitational constant G** changes the proportionality to an equality. In other words:

$$F_g = \frac{Gm_1m_2}{r^2} \qquad (4.1)$$

where

F_g is the magnitude of the gravitational force (in Newtons);

m_1 and m_2 are the masses of the two bodies (in kilograms);

r is the distance between their centers of mass (in meters), as shown in **Figure 4.2**);

G is the universal gravitational constant ($G=6.673 \times 10^{-11}$ $m^3/kg\text{-}s^2$ in SI) (footnote on English units)

The direction of the gravitational force of m_2 acting on m_1 is from m_1 towards m_2. Similarly, the direction of the gravitational force of m_1 acting on m_2 is from m_2 towards m_1 (see **Figure 4.2**).

Because the mass of the earth is at least 20 orders of magnitude greater than the mass of most objects on the planet, the gravitational attraction between any two objects on the planet is negligible relative to the gravitational attraction between either object and the earth. For example, the magnitude of the gravitational force between two average sized apples is a mere 0.000 000 000 064 6 newtons (6.46×10^{-11} newtons)‡, compared with the 0.98 N gravitational force between one apple and the earth§ (**Figure 4.3**). Therefore, the gravitational force exerted by the earth is an important force acting on objects near the earth's surface.

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‡ This force was found using Equation 4.1, with $m_1=m_2$ =mass of apple =0.1 kg, , r=radius of apple+radius of apple=0.050 m +0.050 m =0.100 m

§ This force was found using Equation 4.1, with m_1= mass of apple=0.1 kg, m_2=mass of earth=5.976×10^{24} kg, r=radius of earth+radius of apple=6.371×10^6 m +0.05m =6.371×10^6 m

FIG 4.2

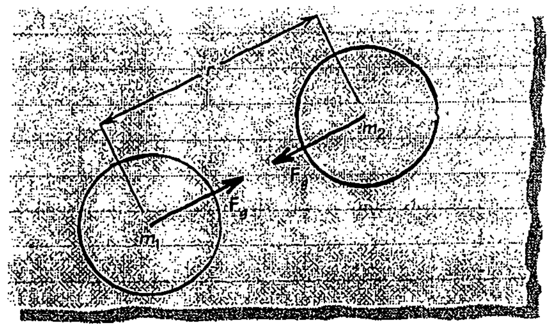

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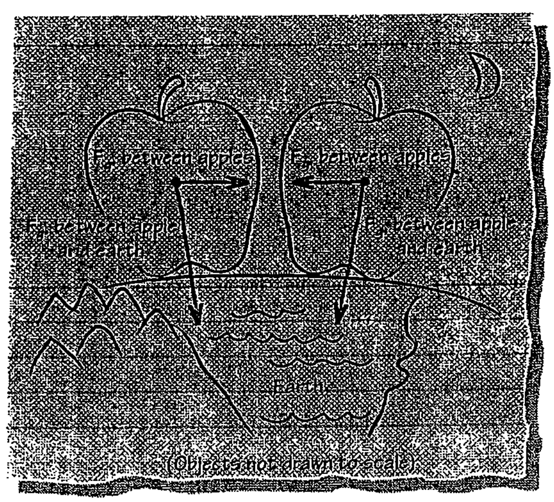
FIG 4.3



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We can simplify Equation 4.1 by realizing that, (a) the mass of the earth is constant, and (b) the distance between the center of mass of the earth and the center of mass of the object can be taken to be the earth's average radius for any object either on the earth's surface or not far above the surface. In other words, once we arbitrarily say that m_1 in Equation 4.1 is the mass of the earth, we can group our three constants---G, m_1, and r^2---into a new constant g. Therefore, Equation 4.1 can be rewritten as:

$$F_g = \frac{Gm_1m_2}{r^2} = m_2\left(\frac{Gm_1}{r^2}\right) = mg \qquad (4.2)$$

where $g = \frac{Gm_1}{r^2}$ and is called the gravitational constant, and m (=m_2) is the mass in kilograms of an object. By substituting the values of G, m_1 for the earth (=5.976×10^{24} kg), and the mean radius of the earth (=6.371×10^6 m) ** into Equation 4.2, we find that near the earth the magnitude of the force of gravity (in newtons) on any object of mass m (in kilograms) is:

$$F_g = m\ 9.807\ (m/s^2) \qquad (4.3A)$$

The magnitude of gravity force in Equation 4.3A is given a special label—the object's **weight on earth** (W_{earth}). W_{earth} is the magnitude of the force of gravity exerted by earth on an object, and is the product of the mass of the object and the gravitational constant g (= 9.807 m/s^2= 32.2 ft/s^2 for the earth):

$$\text{Weight on earth} = W_{earth} = F_g = m\ 9.807\ (m/s^2) \qquad (4.3B)$$

Occasionally you may observe mass and force units seemingly used to mean the same thing. An example is the label on the candy bar sold in the U.S. as shown in **Figure 4.4**, where the net weight is given as 104.9 grams and 3.70 ounces (=0.231 pound). "Gram" is a mass unit and "ounce" is a force unit. M&M*Mars™ (the maker of Snickers) assumes that its candy will be consumed on the earth and is saying "the gravitational force

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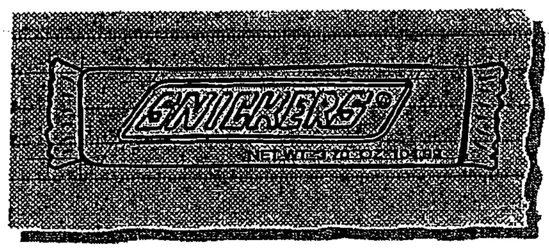
** The earth is not a perfect sphere. It is approximately an ellipsoid, flattened at the poles and bulging at the equator. Its equatorial radius is greater than its polar radius by 21 km (reference to RHK). What this means is that gravitational force is slightly greater at the poles (g=9.835 m/s^2) than at the equator (9.78 m/s^2). For most engineering work, this difference is insignificant and we use a value of "g" based on the mean radius of the earth.

FIG 4.4



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experienced by this 104.9 grams of candy is 3.70 ounces (=1.02 newtons) when they candy is near the earth's surface."

Weight is actually a body force. This means that the gravitational force exists between every atom in the object and every atom in the earth (**Figure 4.5A**). There will be times when this distributed nature of the gravitational force needs to be considered in engineering practice. Formal procedures for incorporating the distributed nature of an object's weight into engineering calculations are addressed in Chapter 8. There will be other times (far more common in the types of problems included in this text and in engineering practice more generally) when it will be sufficient to lump all the distributed gravitational forces acting on an object into a single force. This single force, which represents the weight of the object, is directed from the center of mass of the object towards the center of mass of the earth[††] (**Figure 4.5B**).

Sample Problem #1 Gravity, Weight and Mass

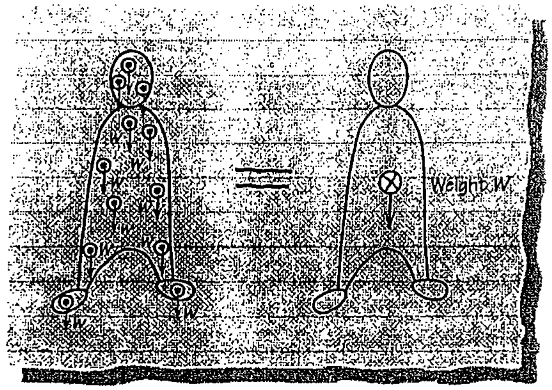
4.3 Contact and Bonding Forces

Contact and bonding forces result from electric and magnetic interactions that are responsible for the bonding of atoms and the structure of solids. Under this general heading is force that prevents one solid object from moving through another solid object (normal contact force), force that results when one solid object slides or tends to slide across another (friction force), force that results from the interaction of a solid object and a fluid (fluid contact pressure), and force that results when molecules within a solid object are pulled relative to one another (tension force), pushed relative to one another (compression force), shifted relative to one another (shear force).

Normal contact force—Whenever two solid objects are in contact with each other, each experiences a force that is *perpendicular* to their two contacting surfaces called **normal contact force**. For example, normal contact force acts on a piano key where a fingertip presses downward on it, and an equal and opposite normal contact force acts on the fingertip where the key presses upward on it **Figure 4.6A**). Similarly, a normal contact force acts on the table where this book sits, and an equal and opposite normal contact force acts on the book where table pushes back (this is Newton's Third Law). Normal

†† The procedure for finding the center of mass is discussed in Chapter 8.

FALL'01, FIG 4.5



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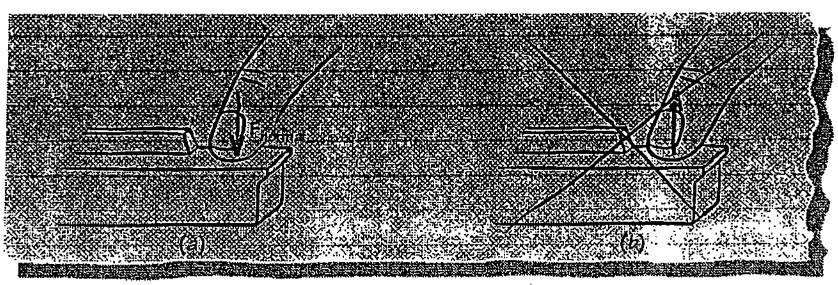

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contact force is directed so as to bring the two solids together. What this means in practical terms is that a clean fingertip contacting a piano key can push but can't pull on the key, as illustrated in **Figure 4.6B**.

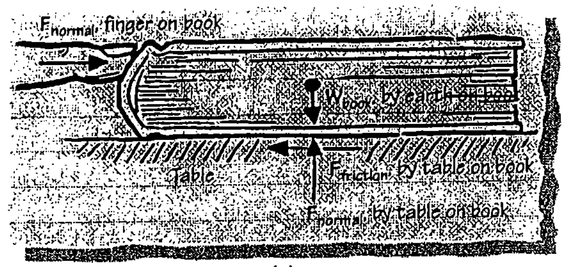
Friction Force— If you attempt to slide a solid object over another solid object, the motion is resisted by interactions between the surfaces of the two objects. This resistance is **friction force** and is oriented *parallel* to their two contacting surfaces in a direction opposite the direction of (pending) motion. For example, if you push on an edge of this book as it rests on a table, as in **Figure 4.7**, the friction force exerted by the table on the book is in the direction opposite to the sliding direction. An equal and opposite friction force acts on the table. Friction force is related to and limited by normal contact force and the characteristics (e.g., smoothness) of the objects in contact. Normal contact force must be present for friction force to be present (but not visa versa).

Fluid contact pressure— As fluid presses on or moves past a solid object it wets the surface of the solid and applies a force to the surface; we call this force the **fluid contact pressure**. (Fluids is the general term for gases and liquids — substances that change shape to fill a volume.) You have probably experienced this sort of force if you have ever put your hand out of the window of a moving car---there is a definite force pushing backwards on your hand. When we refer to the interaction between a fluid and a solid, we will typically be talking in terms of the fluid contact pressure. The dimensions of pressure are force/area, and so pressure units are N/m^2 in the SI system, lbs/in^2 or psi in the U.S. Customary system.

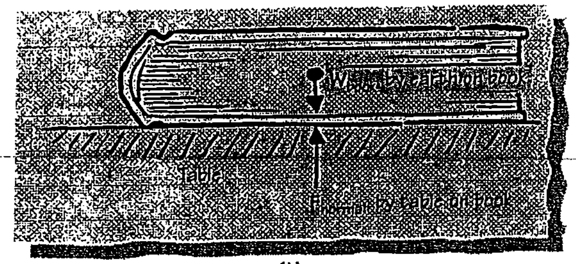
The fluid contact pressures that engineers work with may be very small (e.g., 1000 N/m^2, water pressure on the bottom of a full tea kettle), of medium size (e.g., 500,000 N/m^2 air pressure in a bicycle tire), or very large (e.g., 3,000,000 N/m^2 air pressure in a scuba tank or fluid pressure in a hydraulic brake system).

Tension Force—A cable attached to a solid object and pulled taut is said to be under **tension**. For example, consider a cable holding up a crate, as in **Figure 4.8**. Tension in the cable is transmitted along the cable. Microscopically, each atom of the cable "pulls" on the atom next to it and is in turn pulled by that atom, according to Newton's third law. In this way the force pulling on one end of the cable is transmitted to the object on the other end. If we were to cut the cable at any point and insert a spring scale at the cut ends, the spring scale would read the tension force $\mathbf{F}_{tension}$ directly. Tension forces in systems

FALL '01, FIG 4.7



(a)



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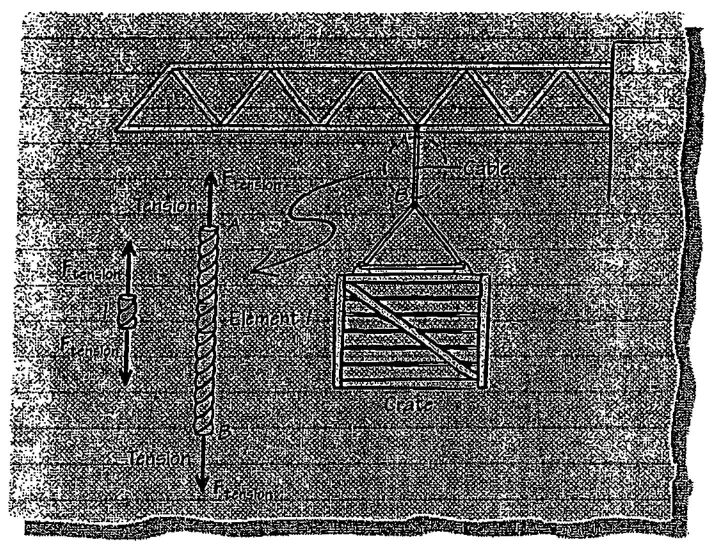
FALL '01, FIG 4.8



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may be very small (e.g., 0.001 N, spider swinging on its web) or very large (e.g., 1,000,000 N *tension in suspension cables of Bay bridge?? need to verify this number*).

Compression Force—When the atoms that make up a solid object are pushed closer together, they experience **compressive force**. For example, consider a vertical column holding up a wooden deck, as in **Figure 4.9**. Compression is transmitted along the column as the deck pushes down from the top and the support pillar pushes up from the bottom. As with the cable in tension described above, successive atoms of the column act on each other. In the case of the column in compression, successive atoms "push" on each other with compression force $\mathbf{F}_{\mathbf{compression}}$. Compression forces in systems may be very small (e.g., 0.5 N compression applied by household tweezers) to very large (e.g., 1,000,000 N compression applied during sheet metal stamping).

Shear Force—When the atoms that make up a solid object are shifted relative to one another, they experience **shear force**. For example, consider a rock climber standing on the small rock toe-hold (**Figure 4.10**). At the interface between the toe-hold and the larger rock mass, shear force is transmitted. Microscopically, atoms on the right of the interface shift downward (ever so slightly) relative to the atoms on the left. This shift results in an upward shear force acting on the toe-hold, and an equal and opposite shear force on the larger rock mass. Notice that the shear force is parallel to the interface.

Summary---Forces come in a wide variety of sizes and types (gravity, normal contact, friction, fluid contact, tension, compression, shear). In performing equilibrium analysis, we look at a physical situation and identify the types of forces present, as discussed in the next section.

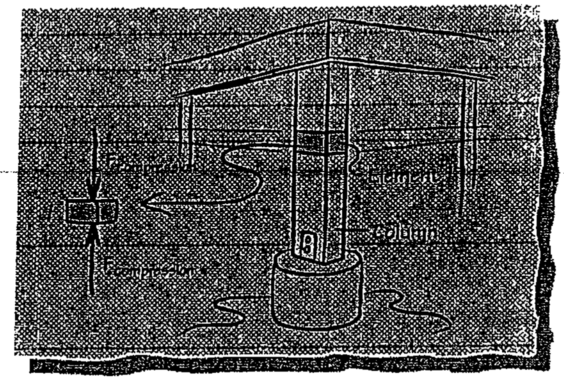
Sample Problem #2 Identifying Types of Forces

4.4 Carrying out analysis

Which forces are important—zooming-in

The ability to identify the relevant portion of the world and the forces acting on that portion is key to being able to address questions concerning structural integrity and system performance. We will refer to the portion as the **system** and to the forces acting ON the system as **external forces**. External forces may be any of the types discussed above—gravity, normal contact, friction, fluid contact pressure, tension, or compression.

FALL '01, FIG 4.9



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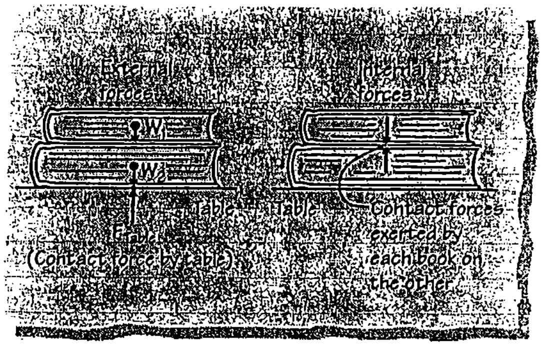
External forces are to be contrasted with **internal forces**, which exist within the system in equal and opposite pairs and are therefore self-canceling (this is Newton's Third Law). For example, if two stacked books resting on a table are considered to be the system (**Figure 4.11A**), the external forces are the weights of the two books (i.e., the gravitational force exerted by the earth on each book) and the normal contact force that the table exerts on the bottom book. The normal contact forces between the two books (the push of the lower book on the upper book and the equal and opposite push of the upper book on the lower book) are internal forces. Because they are equal in magnitude (size) and opposite in direction, they exactly cancel each other. If, on the other hand, we define our system to be the upper book, the external forces acting on the system are the weight of the upper book and the normal contact force of the lower book on the upper book (**Figure 4.11B**). Finally, if we define our system to be the lower book, the external forces acting on this system are the weight of the lower book, and the normal contact forces of the upper book and table acting on the lower book (**Figure 4.11C**). (Notice that Figures 4.11B and 4.10C combine to form the two-book system in Figure 4.11A.)

The presence of external and internal forces is further illustrated in the following example:

Consider a person standing on a ladder that is leaned against a building. The ladder consists of two vertical stringers and eight horizontal rungs that run between the stringers (**Figure 4.12**). Normal contact and friction forces exist between the ladder and the building and between the ladder and the ground. Normal contact and friction forces are also present between the person's hands and feet and the ladder rungs. In addition, there is the gravitational force (weight) of the person and ladder. Do we need to consider all of these forces? The answer depends on what we want to know about the situation. For example:

- Case 1.1: If we want to know whether the feet of the ladder will begin sliding away from the building (not a desirable state of affairs!), we could take the system to be the person and the ladder. The external forces acting on this system are the weights of the person and ladder and the normal contact and friction forces that the wall and ground exert on the ladder (**Figure 4.13**). The normal contact and friction forces between the person and the ladder are internal to our system.
- Case 1.2: Alternately, we could take the ladder alone as our system in determining whether the ladder will slide (**Figure 4.14**). The external forces acting on this system

FALL '01, FIG 4.11



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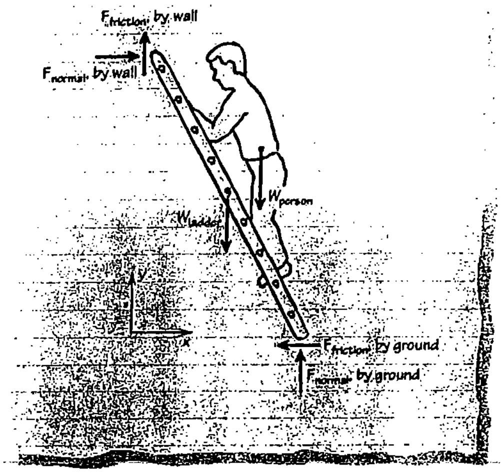

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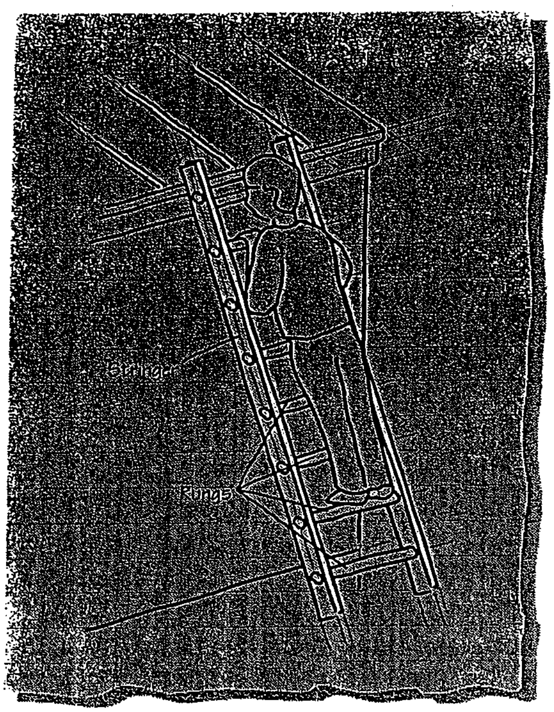

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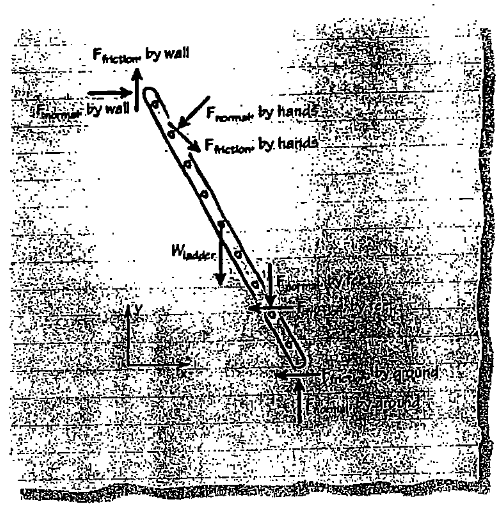

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are the weight of the ladder, the normal contact and friction forces that the person's hands and feet exert on the rungs, and the normal contact and friction forces that the wall and ground exert on the ladder.

- Case 1.3: If we want to know whether the connections between a rung and the stringers are strong enough, we would take the system to be the rung on which the person is standing. The external forces exerted on this rung are the normal contact and friction forces exerted by the person's feet and tension, compression, and shear forces that the stringers apply to the rung (see **Figure 4.15**).
- Case 1.4: If we want to know about the forces exerted on the person's lower back, we could begin by defining the system as the person (Case 1.4A). The external forces exerted on this system are the weight of the person and the normal contact and friction forces exerted by the ladder on the person's hands and feet (**Figure 4.16A**). These external forces are in contrast to forces internal to the person: tension forces created by muscles and normal contact between bones. An analysis of this system would be followed by an analysis of another system---the upper torso of the person (Case 1.4B). The external forces exerted on this system (shown in **Figure 4.16B**) are the weight of the upper torso, the normal contact and friction forces exerted by the ladder on the person's hands, AND normal contact and tension forces exerted by the lower torso on the upper torso. It is these latter external forces that are carried by the lower back. This case illustrates that sometimes we may need to take an iterative approach to analysis, first looking at one system, then redefining the system as a portion of the original system.

The external and internal forces involved with each of these cases are summarized in Table 4.1. Notice that the classification of a force as external or internal depends on the definition of the system or object that we are considering.

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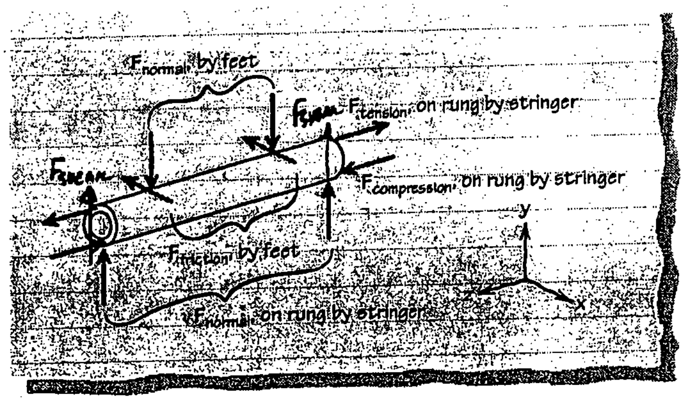

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FALL '01, FIG 4.16

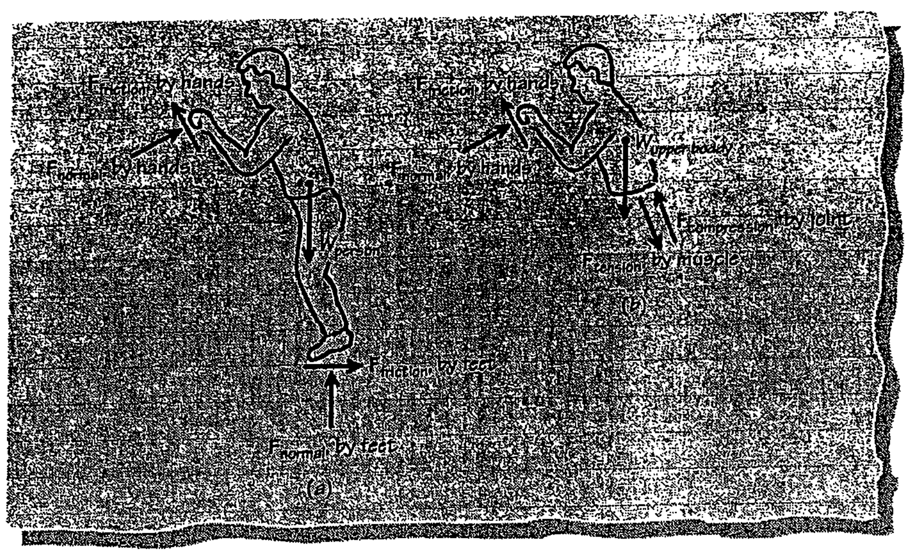

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Table 4.1: Person on a Ladder##
Summary of Forces in Various Situations

Forces:	Case 1.1 *(sketch of system)*	Case 1.2 *(sketch of system)*	Case 1.3 *(sketch of system)*	Case 1.4A *(sketch of system)*	Case 1.4B *(sketch of system)*
Weight of person	E			E	E (only the weight of the upper torso)
Weight of ladder	E	E			
Normal Contact between ladder stringers and roof	E	E			
Friction between ladder stringers and roof	E	E			
Normal Contact between ladder stringers and ground	E	E			
Friction between ladder stringers and ground	E	E			
Normal Contact between person's feet and rungs	I	E	E	E	
Friction between person's feet and rungs	I	E	E	E	
Normal Contact between person's hands and rungs	I	E		E	E
Friction between person's hands and rungs	I	E		E	E
Normal Contact, Tension, Shear and Compression between rungs and ladder stringers	I	I	E		
Tension in Person's Lower Back Muscles	I	I		I	E
Normal Contact in Person's Lower Spine	I	I		I	E

(the gray blocks indicate forces that are not applied to the structure under consideration)
E=External Force; I=Internal Force

In the person-ladder example, we zoomed-in and isolated a portion of the world that is relevant to the particular question we are asking. We called this portion the system. We then identified the forces acting on that system and called these the external forces. It will usually be convenient to define the system so that the forces we are trying to find are external forces. Internal forces come in pairs of equal magnitude and opposite direction and are self-canceling. Therefore, we do not consider them when analyzing the system. Notice that as we went from Case 1.1 to 1.4, forces that were internal to some systems became external to others. Whether a force is external or internal depends on the system of interest.

<u>General Steps for Analysis</u>

The process of zooming-in to define a system, then identifying external forces acting on that system (as we did above) is critical in evaluating the performance of a system. This process is more formally presented in the FIND, GIVEN and ASSUME steps of the Table 1.1 Analysis Procedure.

Zooming-in and identifying external forces then leads to the DRAW step on the procedure. The drawing we create is called a **freebody diagram**---"free" because the boundary cuts the system off from the world around it, "body" because we have defined a specific system to focus on, and "diagram" to emphasize the importance of a visual representation. A freebody diagram shows the external forces that act on the system as vectors (either as resultant forces or as components). Each force is shown on the diagram at its point of application; this is the point on the system where the force acts. Figures 4.13-4.16 are examples of freebody diagrams. We will have a lot more to say about creating freebody diagrams in Chapter 6.

<u>Sample Problem #3 Practice in applying the analysis steps from Chapter 1</u>

4.5 MAGNITUDE AND DIRECTION DEFINE A FORCE

Working with freebody diagrams involves representing and manipulating forces---therefore we now consider how to formally work with the vector quantity of force. Although this section and the next are framed in terms of force, our comments on vectors are equally valid for any vector quantity.

Consider that you are asked to remove a tent stake from the ground, as shown in **Figure 4.17A**. How would you pull on the rope?

It is likely that you would pull so that the rope was aligned with the axis of the stake with somewhere between 100-150 N of force. We can represent the pull force graphically (**Figure 4.17B**). In addition, we can represent the force mathematically, as discussed below.

Magnitude-Angle Representations—Consider a force **F** of known magnitude, where the magnitude is the size of the force and is specified as *F* (we use italics to denote the magnitude of a vector throughout this book).

The direction of this force can be specified by its angular orientation relative to a set of right-handed reference axes. Call these angles θ_x, θ_y, and θ_z, from the x, y and z axes, respectively, as illustrated in **Figure 4.18**, and refer to them as the **direction cosine angles.** They can be specified either in degrees or in radians and can generally be found based on the geometry of the situation. For example, in **Figure 4.19** we are given dimensions related to the cable force that allow us to define the coordinates of two points (A and B) on the force's line of action. Based on these coordinates, we find that:

$$\theta_x = \cos^{-1}[(x_2 - x_1)/L]$$
$$\theta_y = \cos^{-1}[(y_2 - y_1)/L] \qquad (4.4)$$
$$\theta_z = \cos^{-1}[(z_2 - z_1)/L]$$

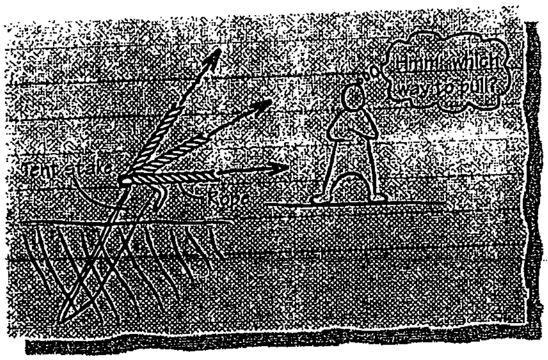
where $L = [(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2]^{0.5}$

When working with direction cosine angles consider that:

1) The angles θ_x, θ_y, and θ_z are not independent of one another. They are related by

FALL'01, FIG 4.17

(B) (A)



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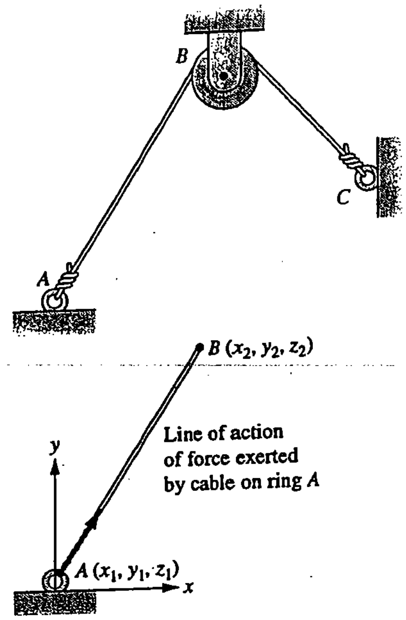
FALL'01, FIG 4.19



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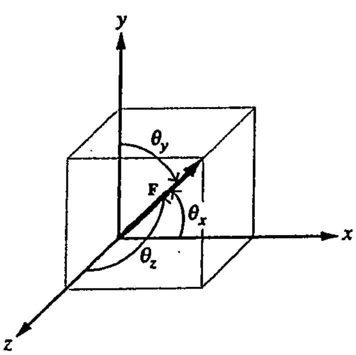
FALL'01, FIG 4.18



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$$[(\cos \theta_x)^2+(\cos \theta_y)^2+(\cos \theta_z)^2]^{0.5}=1 \qquad (4.5)$$

(this expression was proved in Chapter 1; need specific Chapter 1 reference here).

2) If the force is in the plane of two of the reference axes, one of the angles is 90 degrees. For example, if a force is in the xy plane, the angle θ_z between the z axis and the force is 90 degrees. We call a force in the plane of two of the reference axes a **planar or two-dimensional (2D) force**; otherwise it is called a **non-planar or three-dimensional (3D) force**. For planar forces, Equation (4.5) simplifies to:

$$[(\cos \theta_x)^2+(\cos \theta_y)^2]^{0.5}=1 \qquad (4.6)$$

since θ_z =90 degrees and therefore $\cos \theta_z=0$.

Examples of planar and non-planar forces specified with direction cosine angles are shown in **Figures 4.20** and **4.21**, respectively.

3) The direction cosine angles are always defined as positive angles between zero and 360 degrees. Examples of their specification are given in **Figure 4.20** and **4.21.**

An alternative to the direction cosine angle approach described is to use **two angles** (α,β), as illustrated in **Figure 4.22.** The angle α defines the sweep from the x axis to the projection of **F** onto the xy plane (you can think of the projection as the "shadow" that **F** that would be cast on the xy plane, shown as a faint dashed line in **Figure 4.22**). The angle β defines the sweep off the xy plane to **F**. The angles (α,β) can be specified either in degrees or in radians.

When working with (α,β) consider that:

1) If the force is in the xy plane (therefore it is a planar force), β is zero degrees, **Figure 4.23**.
2) The angle α is a positive angle between zero and 360 degrees if defined counter-clockwise relative to the x axis; otherwise it is negative. The angle β is positive if the sweep off the xy plane is towards the +z axis, and is negative is the sweep is towards the –z axis, as shown in **Figures 4.23**, respectively.
3) The angles (α,β) and the direction cosine angles (θ_x, θ_y, θ_z) are related to one another by (proof in a footnote missing):

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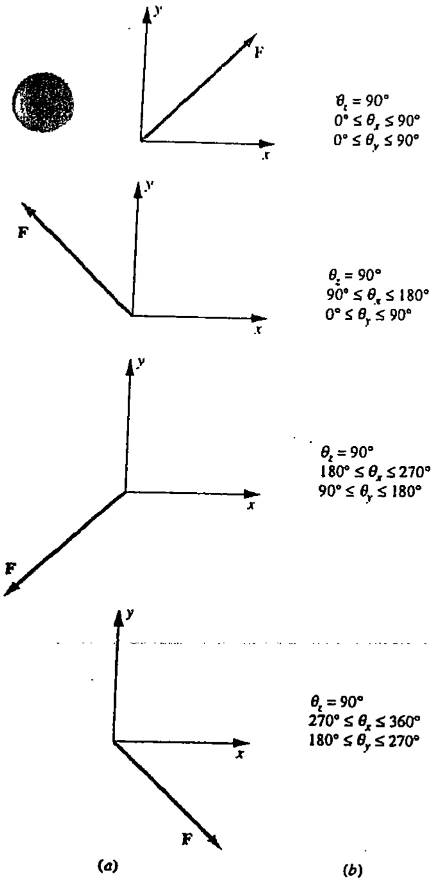

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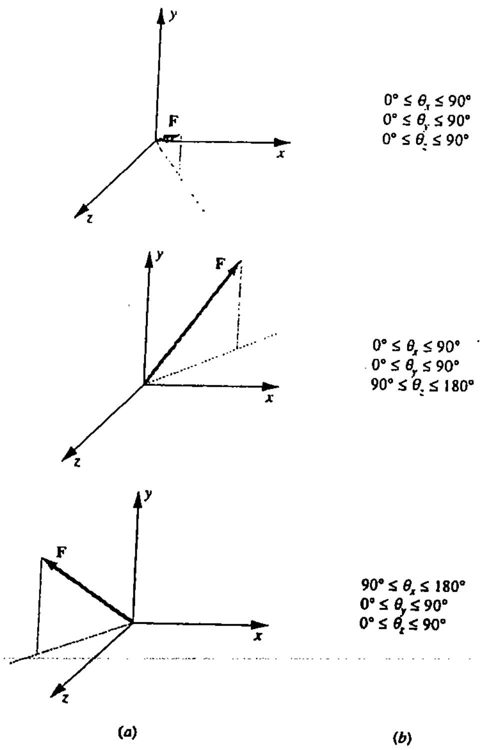
FALL'01, FIG 4.21



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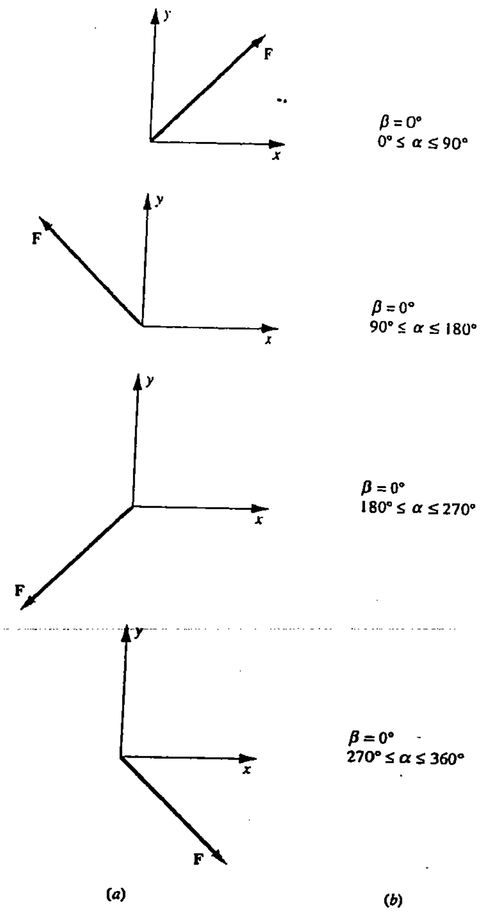
FALL'01, FIG 4.23



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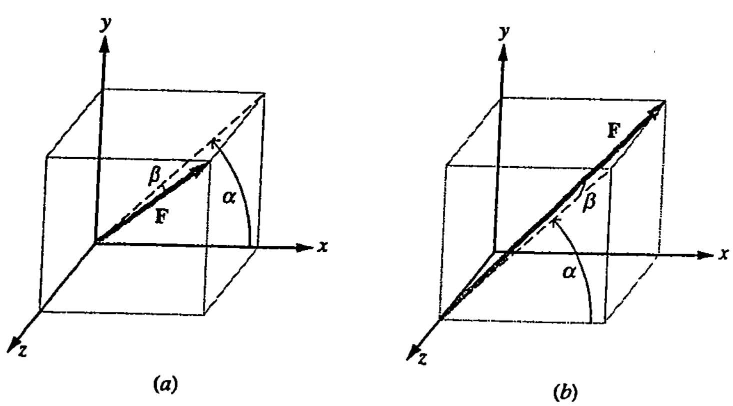
FALL'01, FIG 4.22



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$$\theta_x = \cos[\cos\alpha \cos\beta]^{-1};$$
$$\theta_y = \cos[\sin\alpha \cos\beta]^{-1}; \qquad (4.7A)$$
$$\theta_z = \cos[\sin\beta]^{-1}$$

or

$$\beta = \sin^{-1}[\cos(\theta_z)] \qquad (4.7B)$$
$$\alpha = \tan^{-1}[\cos(\theta_y)/\cos(\theta_x)]$$

Rectangular Component Representation—Another way of representing the magnitude and direction of a force **F** is to specify its three **rectangular components vectors** (**F_x**, **F_y**, **F_z**) relative to a set of reference axes. This is the same thing as specifying the "hike" you would take in x, y, and the z directions to get from the tail to the head of the force vector (**Figure 4.24A**). In other words:

$$\mathbf{F} = \mathbf{F_x} + \mathbf{F_y} + \mathbf{F_z} \qquad (4.8A)$$

This equation can be rewritten in terms of the unit vectors **i**, **j** and **k** (reference to Chapter 1 discussion on unit vectors) as:

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k} \qquad (4.8B)$$

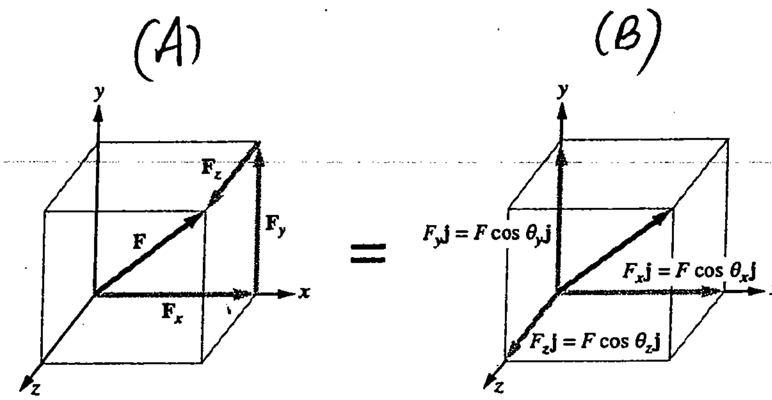
where F_x, F_y and F_z are the projections of **F** onto the x, y and z axes, respectively. We refer to the projections F_x, F_y and F_z as the **scalar components** of **F**. Each scalar component tells us the magnitude of **F** in a particular direction and whether we are "hiking" in the positive or negative direction (so unlike the concept of magnitude, which has no sign associated with it, a scalar component does have a sign associated with it).

If we know the direction cosine angles (θ_x, θ_y, θ_z) and magnitude (*F*) of **F**, we can write the scalar components as:

$$F_x = F\cos\theta_x \qquad (A)$$
$$F_y = F\cos\theta_y \qquad (B) \qquad (4.9)$$
$$F_y = F\cos\theta_z \qquad (C)$$

See **Figure 4.24B**.

FALL'01, FIG 4.24



$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y + \mathbf{F}_z$ (Eq. 4.8A) = $\mathbf{F}_x\mathbf{i} + \mathbf{F}_y\mathbf{j} + \mathbf{F}_z\mathbf{k}$ (Eq. 4.8B) $= F \cos \theta_x \mathbf{i} + F \cos \theta_y \mathbf{j} + F \cos \theta_z \mathbf{k}$ (Eq. 4.10A)

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We now write **F** in terms of its component vectors by substituting from Equation 4.9 into Equation 4.8B:

$$\mathbf{F} = F\cos\theta_x\mathbf{i} + F\cos\theta_y\mathbf{j} + F\cos\theta_z\mathbf{k} \qquad (4.10A)$$

which can be rearranged as:

$$\mathbf{F} = F\ (\cos\theta_x\mathbf{i} + \cos\theta_y\mathbf{j} + \cos\theta_z\mathbf{k}) \qquad (4.10B)$$

The term $(\cos\theta_x\mathbf{i} + \cos\theta_y\mathbf{j} + F\cos\theta_z\mathbf{k})$ in this expression defines a unit vector (reference to Chapter 1 on direction cosine), which we will call **n**. It is aligned with the line of action of **F**. Therefore Equation 4.10B can be rewritten in terms of **n** as:

$$\mathbf{F} = F\ (\cos\theta_x\mathbf{i} + \cos\theta_y\mathbf{j} + \cos\theta_z\mathbf{k}) = F\,\mathbf{n} \qquad (4.11)$$

The discussion up to now on rectangular component representation has been in terms of decomposing **F** into its component vectors $\mathbf{F_x}$, $\mathbf{F_y}$, and $\mathbf{F_z}$. Just as common in analysis is the need to combine the component vectors $\mathbf{F_x}$, $\mathbf{F_y}$, and $\mathbf{F_z}$ into their resultant force **F**. The scalar components can be combined to determine the magnitude of the resultant (F):

$$F = (F_x^2 + F_y^2 + F_z^2)^{0.5} \qquad (4.12)$$

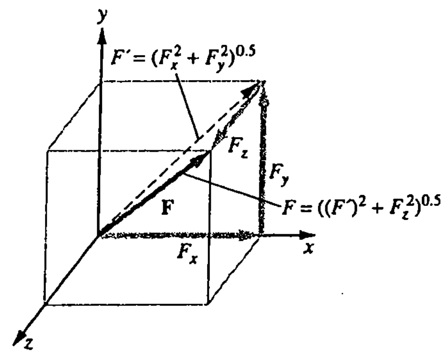
which is found by noting in **Figure 4.25** that $F = (F'^2 + F_z^2)^{0.5}$, where $F' = (F_x^2 + F_y^2)^{0.5}$. Therefore, F the magnitude of **F** is equal to the positive square root of the sum of the squares of its scalar components.

It is also common in analysis to use the scalar components F_x, F_y and F_z of a force to find the direction cosines of the line of action of the force. Rearranging the expressions in 4.9 we find:

$$\cos\theta_x = F_x/F$$
$$\cos\theta_y = F_y/F \qquad (4.13)$$
$$\cos\theta_z = F_z/F$$

where F is given in Equation 4.12. The relationships given in Equations 4.13 are shown graphically in **Figure 4.26.**

FALL '01, FIG 4.25



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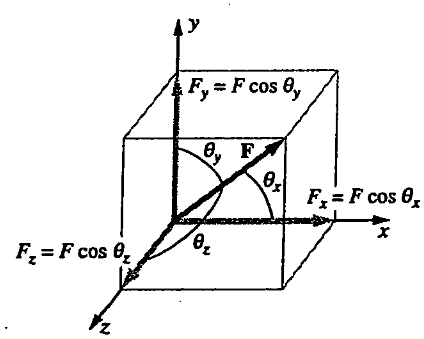
FALL '01, FIG 4.26



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The discussion of rectangular representation of forces has been in terms of non-planar forces. If, on the other hand, a force is planar, Equations 4.8B and 4.10A can be simplified. For example, if the force is contained in the xy plane, F_z=0, θ_z =90 degrees, and cos θ_z =0. Therefore, Equations 4.8B and 4.10A simplify to

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} \qquad (4.14A)$$

(planar version of 4.8B)

$$\mathbf{F} = F \cos\theta_x \mathbf{i} + F \cos\theta_y \mathbf{j} \qquad (4.14B)$$

(planar version of 4.10A)

as illustrated in **Figure 4.27**.

You'll use Equations 4.4-4.14 in various ways as you work with forces and freebody diagrams. Sometimes the force components will be known, and you will be interested in finding the magnitude of the resultant force (so Equation 4.12 will come in handy). Other times, the force direction and magnitude will be known, and you will need to find the components (using Equation 4.9). What this all means is that you need to feel comfortable manipulating forces and their components. If you understand the principles behind the various ways of representing a force, finding magnitudes and directions will become straightforward with a little practice.

Summary To work with forces we need to be concerned with their magnitude and direction. Information about magnitude and direction can be specified using magnitude and angles (either the direction cosine angles, or the α,β angles) or with scalar components, as summarized in **Table 4.2.**

Sample Problem #4 Representing Planar Forces
Sample Problem #5 Representing Non-planar Forces

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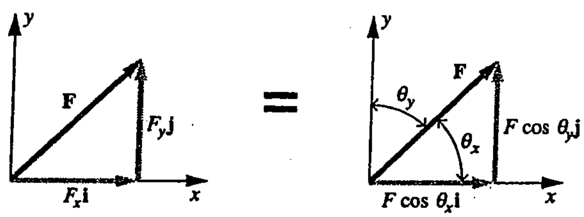

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Table 4.2: Summary of Force Representations

	REPRESENTATION				
	Magnitude & Direction Cosine Angles		Magnitude & (α,β)		Rectangular Components
Non-planar Force (**F**) General Force	*Magnitude:* $F=(F_x^2+F_y^2+F_z^2)^{0.5}$	*Direction:* $\theta_x = \cos^{-1}(F_x/F)$ $\theta_y = \cos^{-1}(F_y/F)$ $\theta_z = \cos^{-1}(F_z/F)$ Furthermore: $(\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z)^{0.5} = 1$ Unit vector **n** in direction of force is: $\mathbf{n} = \cos\theta_x\,\mathbf{i} + \cos\theta_y\,\mathbf{j} + \cos\theta_z\,\mathbf{k}$	*Magnitude:* $F=(F_x^2+F_y^2+F_z^2)^{0.5}$	*Direction:* α; angle from x axis to projection of force onto xy plane. Positive if counter-clockwise. β; angle from xy plane to force vector. Positive if towards +z axis.	Scalar components in the x, y, and z directions are (F_x, F_y, F_z). In vector form: $\mathbf{F}=F_x\mathbf{i}+F_y\mathbf{j}+F_z\mathbf{k}$ Can also be written in terms of magnitude *F* of **F** and direction cosines as: $\mathbf{F}=F\cos\theta_x\,\mathbf{i}+F\cos\theta_y\mathbf{j}+F\cos\theta_z\mathbf{k}$
Planar Force (**F**) Force in the xy reference plane	*Magnitude:* $F=(F_x^2+F_y^2)^{0.5}$, since $F_z=0$	*Direction:* $\theta_x = \cos^{-1}(F_x/F)$ $\theta_y = \cos^{-1}(F_y/F)$ $\theta_z = 90°$ Furthermore: $(\cos^2\theta_x + \cos^2\theta_y)^{0.5} = 1$ Unit vector **n** in direction of force is: $\mathbf{n} = \cos\theta_x\,\mathbf{i} + \cos\theta_y\,\mathbf{j}$	*Magnitude:* $F=(F_x^2+F_y^2)^{0.5}$	*Direction:* α; angle from x axis to projection of force onto xy plane. Positive if counter-clockwise β=0; force is in the x-y plane.	Scalar components in the x, y, and z directions are (F_x, F_y, F_z). In vector form: $\mathbf{F}=F_x\mathbf{i}+F_y\mathbf{j}$ Can also be written in terms of magnitude *F* of **F** and direction cosines as: $\mathbf{F}=F\cos\theta_x\,\mathbf{i}+F\cos\theta_y\mathbf{j}$

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