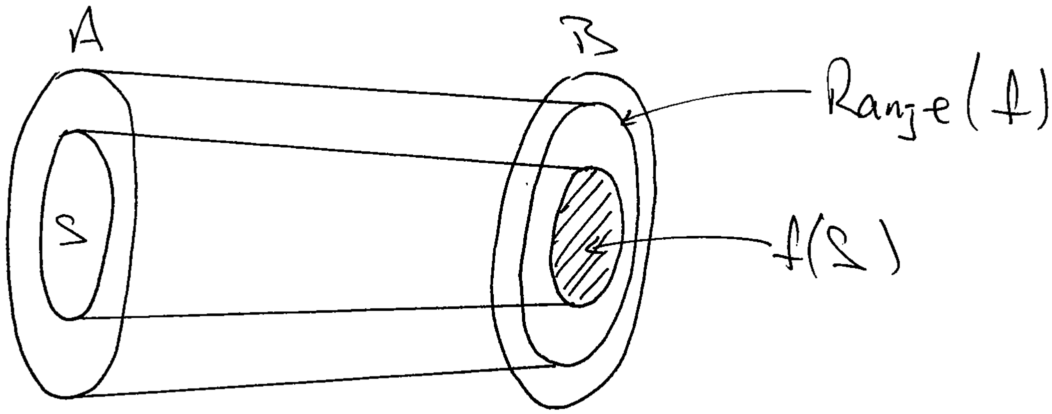
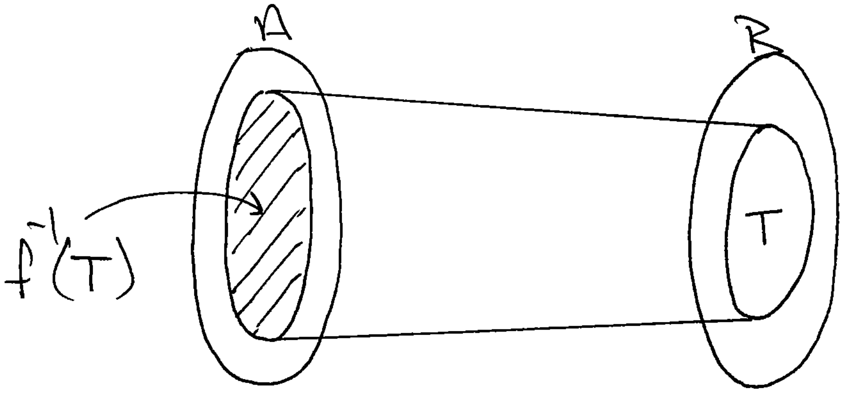
DEFN:

LET $f: A \to B$, $T \subseteq B$. THE PREIMAGE OF T UNDER f IS

$$f^{-1}(T) = \{ x \in A \mid f(x) \in T \}$$





Ex. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$

$S = \{ x \mid 2 \le x \le 3 \}$, $f(S) = \{ y \mid 4 \le y \le 9 \}$

$T = \{ y \mid 5 \le y \le 6 \}$, $f^{-1}(T) = \{ x \mid \sqrt{5} \le x \le \sqrt{6} \}$.

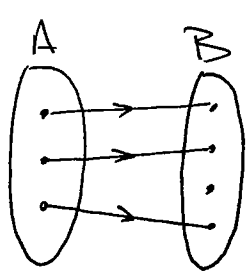
DEFN.

A FUNCTION $f: A \to B$ IS CALLED ONE-TO-ONE OR INJECTIVE IFF

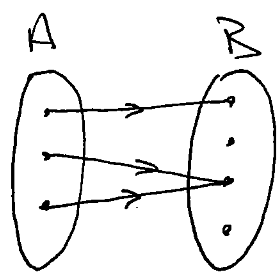
$$\forall x_1, x_2 \in A : f(x_1) = f(x_2) \to x_1 = x_2 .$$

EQUIVALENTLY:

$$\forall x_1, x_2 \in A : x_1 \neq x_2 \to f(x_1) \neq f(x_2)$$



ONE-TO-ONE



NOT ONE-TO-ONE

EX $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$. NOT INJECTIVE

EX $g: \mathbb{R} \to \mathbb{R}$, $g(x) = 2x+3$. INJECTIVE

DEFN.

WE SAY $f: \mathbb{R} \to \mathbb{R}$ IS STRICTLY INCREASING IFF $x < y \to f(x) < f(y)$, AND STRICTLY DECREASING IFF $x < y \to f(x) > f(y)$.

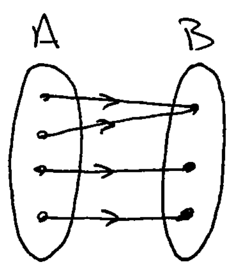
THEOREM

IF $f: \mathbb{R} \to \mathbb{R}$ IS EITHER STRICTLY INCREASING OR STRICTLY DECREASING THEN f IS ONE-TO-ONE.

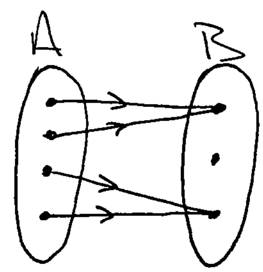
DEFN

$f: A \to B$ is called <u>ONTO</u> or <u>SURJECTIVE</u> if Range $(f) = B$, i.e.

$$\forall y \in B \; \exists x \in A : \; y = f(x)$$



ONTO



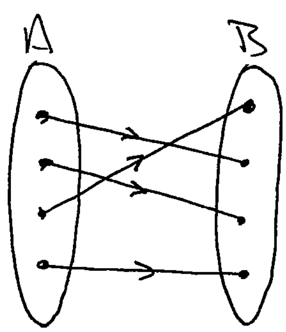
NOT ONTO

EX. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ is NOT ONTO

EX. $g: \mathbb{R} \to \mathbb{R}$, $g(x) = 2x+3$ is ONTO.

DEFN

$f: A \to B$ is called A <u>ONE-TO-ONE CORRESPONDENCE</u> or <u>BIJECTIVE</u> if f is BOTH ONE-TO-ONE AND ONTO.



EX $g(x) = 2x+3$ is BIJECTIVE.

DEFN

THE COMPOSITION OF FUNCTIONS $f: A \to B$ AND $g: B \to C$ IS THE FUNCTION $g \circ f: A \to C$ GIVEN BY:

$$g \circ f(x) = g(f(x)) \quad \forall x \in A$$

EX. LET $A = B = C = \mathbb{R}$, $f(x) = x^2$, $g(x) = 2x + 3$. THEN $g \circ f(x) = g(x^2) = 2x^2 + 3$ AND $f \circ g(x) = f(2x+3) = (2x+3)^2$.

NOTE: IN GENERAL $f \circ g \neq g \circ f$, EVEN WHEN BOTH COMPOSITIONS ARE DEFINED.

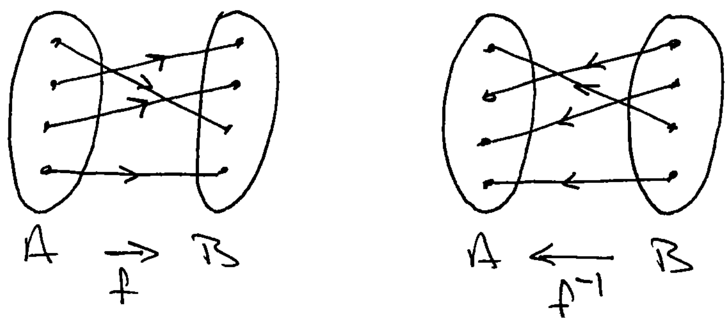
DEFN

LET $f: A \to B$ BE BIJECTIVE. THE INVERSE FUNCTION $f^{-1}: B \to A$ IS THE FUNCTION WHICH ASSIGNS TO EACH ELEMENT $y \in B$ THE UNIQUE ELEMENT $x \in A$ SUCH THAT $y = f(x)$.

NOTE: SUCH AN ELEMENT $x \in A$ EXISTS SINCE f IS ONTO, AND IT IS UNIQUE SINCE f IS ONE-TO-ONE.

ALSO NOTE: $\forall x \in A : f^{-1} \circ f(x) = x$

$\forall y \in B : f \circ f^{-1}(y) = y$.



ALSO NOTE: IF f IS BIJECTIVE THEN SO IS f^{-1}, AND $(f^{-1})^{-1} = f$.

EX. $g(x) = 2x+3$, $g^{-1}(x) = \frac{x-3}{2}$

A FUNCTION WHICH HAS AN INVERSE IS CALLED INVERTIBLE. NOTE f IS INVERTIBLE IFF IT IS BIJECTIVE.

EX. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ IS NON-INVERTIBLE SINCE IT IS NOT BIJECTIVE.

EX. $f: \mathbb{R}^+ \to \mathbb{R}^+$, $f(x) = x^2$ IS BIJECTIVE, SO IS INVERTIBLE. $f^{-1}(x) = \sqrt{x}$.

HERE $\mathbb{R}^+ = \{x \in \mathbb{R} \mid x \geq 0\}$.

FLOOR AND CEILING FUNCTIONS

GIVEN $x \in \mathbb{R}$ WE DEFINE $\lfloor x \rfloor$ AND $\lceil x \rceil$ TO BE THE UNIQUE INTEGERS SUCH THAT

$$x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1$$

i.e. $\lfloor x \rfloor$ = GREATEST INTEGER WHICH IS LESS THAN OR EQUAL TO x

$\lceil x \rceil$ = LEAST INTEGER WHICH IS GREATER THAN OR EQUAL TO x.

EQUIVALENTLY:

$$N = \lfloor x \rfloor \quad \text{IFF} \quad N \leq x < N+1$$

AND

$$N = \lceil x \rceil \quad \text{IFF} \quad N - 1 < x \leq N$$