

## Quiz I Key

1. Use Truth tables to show that  $(p \oplus q) \vee (p \rightarrow q)$  is a tautology.

| p | q | $(p \oplus q)$ | $(p \rightarrow q)$ | $(p \oplus q) \vee (p \rightarrow q)$ |
|---|---|----------------|---------------------|---------------------------------------|
| T | T | F              | T                   | T                                     |
| T | F | T              | F                   | T                                     |
| F | T | T              | T                   | T                                     |
| F | F | F              | T                   | T                                     |

No combination of p and q results in False, thus  $(p \oplus q) \vee (p \rightarrow q)$  is a tautology.

2. Use known equivalences to show that  $(p \wedge q) \rightarrow (p \rightarrow q)$  is a tautology.

$$\begin{aligned}
 & (p \wedge q) \rightarrow (p \rightarrow q) \\
 \equiv & \neg(p \wedge q) \rightarrow (\neg p \vee q) && \text{Equivalence to Logic} \\
 \equiv & (\neg p \vee \neg q) \vee (\neg p \vee q) && \text{De Morgan's 1st Law} \\
 \equiv & \neg p \vee \neg q \vee \neg p \vee q && \text{Associativity Law} \\
 \equiv & \neg p \vee \neg p \vee \neg q \vee q && \text{Commutativity Law} \\
 \equiv & (\neg p \vee \neg p) \vee (\neg q \vee q) && \text{rewrite} \\
 \equiv & (\neg p \vee \neg p) \vee T && \text{Negation Law} \\
 \equiv & \neg p \vee T && \text{Idempotent Law} \\
 \equiv & T && \text{Domination Law}
 \end{aligned}$$

3. Let s and p be the propositions "I study hard" and "pass the exam".

(a) Translate the compound proposition " $\neg(s \rightarrow p) \wedge \neg(\neg s \rightarrow \neg p)$ " into an English sentence.

$$\begin{aligned}
 & \neg(s \rightarrow p) \\
 & \text{if I study, then I won't pass the exam.} \\
 & \neg(\neg s \rightarrow \neg p) \equiv \neg(p \rightarrow s) \\
 & \text{if I slack, I won't fail.}
 \end{aligned}$$

Answer: If I study hard then I won't pass the exam, and if I pass the course then I didn't study hard.

OR

If I study, then I won't pass, but if I slack, I won't fail.

(b) Translate the following sentence into a compound proposition "I pass the exam only if I study hard, and I fail the exam means I did not study hard".

Using the following conventions,

p – pass the exam

s – study hard

We may write the compound proposition as:

$$(p \rightarrow s) \wedge (\neg p \rightarrow \neg s) \equiv p \leftrightarrow s$$

This was the intended correct answer. However, there was some confusion as to whether “only if” mean “if and only if”. Since this was difficult to differentiate from an equivalence, I also accepted:

$$(p \leftrightarrow s) \wedge (\neg p \rightarrow \neg s)$$

Point of note: there has been some confusion concerning whether it should not have been  $(s \rightarrow p)$  as the first part. However, the use of “only if” is not the same as the use of “if”.

4. Suppose the domain of  $x$  and  $y$  is all integers. Suppose the predicate  $Q(x,y)$  means “ $x + 2y = 5$ ”. What are the truth values of the following statements? Please explain.

(a)  $\forall y Q(1,y)$

False – only true for  $y = 2$ .

(b)  $\forall y \exists x Q(x,y)$

True – set  $x = 5 - 2y$  for any given  $y$ , and you have your  $x$ .

5. Using basic logical equivalences to show that  $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$ .

$$\begin{aligned} & \neg p \rightarrow (q \rightarrow r) \\ \equiv & \neg p \rightarrow (\neg q \vee r) && \text{Equivalence to Logic} \\ \equiv & p \vee (\neg q \vee r) && \text{Equivalence to Logic} \\ \equiv & (\neg q \vee r) \vee p && \text{Commutativity of Disjunctions} \\ \equiv & \neg q \vee (r \vee p) && \text{Associative Property} \\ \equiv & q \rightarrow (r \vee p) && \text{Logic to Equivalence} \end{aligned}$$

Thus  $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$