Midterm Key

Discrete Math CMPE 016

Fall Quarter 2004

Problem #12

1. (10 points) $\sum_{i=2}^{n}[2(i-1)^2 + 3^{2i-1}] =$

   Solution:

   First, break the summation down and two pieces:

$$\sum_{i=2}^{n}[2(i-1)^2 + 3^{2i-1}] = \sum_{i=2}^{n}2(i-1)^2 + \sum_{i=2}^{n}3^{2i-1}$$

   Now, solve each piece separately:

$\sum_{i=2}^{n}2(i-1)^2$	First Piece
$= \sum_{i=1}^{n-1}2[(i+1)-1]^2$	Reset indices
$= \sum_{i=1}^{n-1}2i^2$	Simplify
$= 2\sum_{i=1}^{n-1}i^2$	Pull constant out of summation
$= 2[\frac{(n-1)((n-1)+1)(2(n-1)+1)}{6}]$	Note that $\sum_{k=1}^{n}k^2 = \frac{n(n+1)(2n+1)}{6}$
$= 2[\frac{(n-1)n(2n-1)}{6}]$	Simplify
$= \frac{(2n^2-2n)(2n-1)}{6}$	Distribute 2 and n through and simplify
$= \frac{4n^3-2n^2-4n^2+2n}{6}$	Expand Quadratic
$= \frac{4n^3-6n^2+2n}{6}$	Simplify
$= \frac{2}{3}n^3 - n^2 + \frac{1}{3}n$	Simplify and separate fractions

$\sum_{i=2}^{n} 3^{2i-1}$	Second piece
$= \sum_{i=0}^{n-2} 3^{2(i+2)-1}$	reset indices
$= \sum_{i=0}^{n-2} 3^{2i+3}$	simplify power
$= \sum_{i=0}^{n-2} (3^2)^i 3^3$	power manipulations
$= \sum_{i=0}^{n-2} 9^i 27$	simplify
$= 27\sum_{i=0}^{n-2} 9i$	Pull out the constant
$= 27[\frac{9^{(n-2)+1}-1]}{9-1}]$	Note that $\sum_{i=0}^{n} a^i = \frac{a^{i+1}-1}{a-1}$
$= 27[\frac{9^{n-1}-1}{8}$	Simplify
$= \frac{27*9^{n-1}}{8} - \frac{27}{8}$	Distribute 27 through and separate fractions
$= \frac{3*9^n}{8} - \frac{27}{8}$	Pull a 9 out of 27 and place it in 9^{n-1}, incrementing the power

Now put the pieces together:

$$\sum_{i=2}^{n} [2(i-1)^2 + 3^{2i-1}] = \sum_{i=2}^{n} 2(i-1)^2 + \sum_{i=2}^{n} 3^{2i-1} = \frac{2}{3}n^3 - n^2 + \frac{1}{3}n + \frac{3*9^n}{8} - \frac{27}{8}$$

Which is the final answer.