

CMPE 16. Applied Discrete Math

Review (12/4/03)

Chapter 1.

Logic and proof: Section 1.1-1.5

Proposition, Operations of propositions ($\neg, \wedge, \vee, \rightarrow, \leftrightarrow, \oplus$), negation, and, or, implication, biconditional, exclusive or): **Translation English to proposition and vice versa**

Truth table: **Truth table of proposition**

Tautology, contradiction, contingency: **Determine contradictions and tautology**

Logically equivalent: $p \leftrightarrow q$ is a tautology, to verify this, try all truth values for the variable in p and q and show that p and q has the same truth values

Some logical equivalences: Laws in 1.2 Table 5: **Use existing equivalences to prove other equivalences**

Predicate – propositional function, the truth value is determined after the truth values of all variables are given.

Quantifier: universe of discourse, existential quantifier \exists and universal quantifier \forall .

Translate English to (nested) quantifications and vice versa
Determine the truth values of (nested) quantifications

Rules of inference: 1.5 Table 1

Translate English into proposition, derive conclusions using rules of inference.

Methods of Proving Theorem

Direct proof, indirect proof, proof by contradiction, proof by cases

Sets: Section 1.6-1.7

Set, set builder, empty set (Null set), Venn diagram, subset, proper subset, Cartesian product

Set operations: union, intersection, difference, complement, Set identities(1.5 Table 1

Set builder and basic concepts and operators
Use Venn diagram to prove equality

Functions: Section 1.8

Function, domain, image, one-to-one, onto, bijection/one-to-one correspondence, inverse function, floor function, ceiling function

Determine if a function is onto, one-to-one, and bijection

Chapter 2.

2.4 Division: $a|b$, divisibility, primes and composite, verify a prime, dividend, divisor, quotient, residual, gcd, pairwise primes, least common multiple, modular arithmetic, congruent

Prime factorization, gcd, lcm using factorization;

2.5 (binary, octal, hexadecimal) \leftrightarrow decimal, binary operation
Euclidean algorithm for finding gcd

Chapter 3.

3.1 Backward reasoning, counterexample, proof by construction

3.2 Sequence and Summations: **solve** $(\sum_{i=m}^n ai^2 + bi + c, \sum_{i=m}^n ar^i + b, \sum_{i=1}^n \sum_{j=1}^m (ai + br^i + cj + dr^i + e))$

3.3 Mathematical induction

Step 1: What is the proposition $P(n)$

Step 2: Basis step: show that $P(0)$ is true

Step 3: Inductive step: show that $P(n+1)$ is true if $P(n)$ is true

Strong mathematical Induction.

Four types of problem: summation of polynomials or powers, divisibility and modular arithmetic, inequality, sets

3.4 Recursive definitions

The basis value and the inductive definition,

Evaluation and derivation of recursive definitions.

Recursive definition \leftrightarrow normal function definition

Chapter 4,5.

4.1 Counting basics

The product rule, the sum rule, combo

4.3 Permutation and combinations

Formulae for permutation and combination, $c(n+1, r) = c(n, r) + c(n, r-1)$, Pascal's Identity

Problems: cards, passwords, bit strings, tennis team members, etc

5.1 Discrete probability

Solving two counting problems: $|E|$ and $|S|$, use permutation or combinations on both

Problems: cards, passwords, bit strings, tennis team members, etc

4.2 Pigeon hole principle

The generalized principle: k objects, m bins, there are at least $\text{ceiling}(k/m)$ objects in one of the bins.

Two types problem: (1) known k, m , find how many objects in one bin

(2) known $\text{ceiling}(k/m)$ and m , how many objects are required.

Chapter 6.

6.1 Recurrence relations

Rabbits and the Fibonacci numbers, the tower of Hanoi, compound interest

6.2 Solving recurrence relations

Roots of characteristic equation, linear combination of the powers of the roots, solving for the linear coefficients from the initial condition