CMPE 016. Applied Discrete Math
Quiz No. 3. (11/20/2003)

Name: ___________________________ Student Number: _______-______-_________.

1. What is the value of the sum $\sum_{i=1}^{n}(3^i - 5i + 20)$ (20 points)

$$\sum_{i=1}^{n}(3^i - 5i + 20)$$
$$= \sum_{i=1}^{n}(3^i) + \sum_{i=1}^{n}(-5i) + \sum_{i=1}^{n}(20)$$
$$= (\frac{3^{n+1}-1}{3-1} - 1) - 5\frac{(n+1)n}{2} + 20n$$
$$= (\frac{3^{n+1}}{2} - 3/2) - \frac{5n^2+5n}{2} + \frac{40n}{2}$$
$$= \frac{3^{n+1} - 5n^2 + 35n - 3}{2}$$

2. Use mathematical induction to prove $\sum_{i=1}^{n}(8i+2) = 4n^2 + 6n$ (20 points)

Basic Step: The equation is true for n=1 because the left side is 10 and the right side is 4+6=10

Induction Step: If P(k) is true, that means $\sum_{i=1}^{k}(8i+2) = 4k^2 + 6k$, we need to prove P(k+1) is true, that means $\sum_{i=1}^{k+1}(8i+2) = 4(k+1)^2 + 6(k+1)$

Notice that for the left side, the difference between P(k) and P(k+1) is 8(k+1)+2=8k+10

For the right side, the difference between P(k) and P(k+1) is $4(k+1)^2 + 6(k+1) - 4k^2 - 6k$
$= 8k + 4 + 6 = 8k + 10$

The differences between P(k) and P(k+1) on both sides are the same, therefore, if P(k) is true, P(k+1) is also true.

3. Use mathematic induction to prove $99n < n^3$ for integer $n \ge 10$ (20 points)

Basic step: for n=10, 990<1000, therefore true.
Induction step: The difference between P(k) and P(k+1) for the left side is 99(k+1)-99k=99
The difference between P(k) and P(k+1) on the right side is (k+1) $(k+1)^3 - k^3 = 3k^2 + 3k + 1$
This is larger than 99 when $n \ge 10$.
Therefore, if P(k) is true, then P(k+1) is also true.

4. Use mathematic induction to prove $3 | 4^n - 1$ (20 points)

Basic step: $3 | 4^1 - 1$ is true
Induction step: if we assume $3 | 4^k - 1$, we want to prove that $3 | 4^{k+1} - 1$
Notice that the difference between $4^{k+1} - 1$ and $4^k - 1$ is $3 \cdot 4^k$, which is divisible by 3,
Therefore, if $3 | 4^k - 1$, then $3 | 4^{k+1} - 1$.