

CMPE 016. Applied Discrete Math  
Quiz No. 2. (10/23/03)

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Name: \_\_\_\_\_ Student Number: \_\_\_\_\_ - \_\_\_\_\_ - \_\_\_\_\_.

1. Suppose the domain of  $x$  includes all the teapots in a kitchen and the domain of  $y$  includes all the lids in the kitchen. Suppose the predicate  $P(x,y)$  means a lid  $y$  fits a teapot  $x$ .

(a) Translate the following sentence into a proposition using quantifiers, predicate, and logical operators: “If there is a lid that fits all teapots then for every teapot there exists a lid that fits it” (10 points).

$$(\exists y \forall x P(x,y)) \rightarrow (\forall x \exists y P(x,y))$$

(b) Translate the following proposition into an English sentence:

$$\forall y \exists x \exists z (x \neq z \wedge P(x,y) \wedge P(z,y)) \quad (10 \text{ points})$$

Every lid fits at least two teapots

2. What is the truth value of the statement " $\forall x \exists y (x \cdot (y+0.1)=0)$ " if the domains of  $x$  and  $y$  are real numbers, please explain. (truth value: 3 points; explanation: 7 points) What if the domains of  $x$  and  $y$  are integers, please explain. (truth value: 3 points; explanation: 7 points)

When the domains are real numbers, it is true. Because for every  $x$ , if  $y=-0.1$ , then it is true.

When the domain are integers, it is false. Because for example, for  $x=1$ , no matter what  $y$  is,  $y+0.1$  will not be 0, therefore,  $x(y+0.1)$  will not be 0.

3. Let  $A=\{1,2,3,4\}$  and  $B=\{x \mid x \text{ is an integer and } 2 \leq x \leq 6\}$ . Find (a)  $A \cup B$  (b)  $A \cap B$  (c)  $A - B$  (d) All the subsets of  $A$ . (20 points, 5 points each)

Solution:

(a)  $A \cup B = \{1,2,3,4,5,6\}$

(b)  $A \cap B = \{2,3,4\}$

(c)  $A - B = \{1\}$

(d) All the subsets of  $A$ :

$\emptyset$ ,

$\{1\}, \{2\}, \{3\}, \{4\}$ ,

$\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}$ ,

$\{2,3,4\}, \{1,3,4\}, \{1,2,4\}, \{1,2,3\}$ ,

$\{1,2,3,4\}$

4. Let A, B, and C be sets. Use Venn diagram to show that  $(B-A) \cap (A-C) = \emptyset$ . Please explain. (15 points, Venn diagram 10 points, explanation 5 points)

Solution:

The Venn diagram:

