CMPE 16. Applied Discrete Math
Final Exam (12/09/03)

Name: ____________________________ Student Number: _______-_____-_________.

Problem No.	Points	Scores
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10+(10)	
Total	110+(10)	

1. (a) Use a truth table to show that $\neg(p \to q)$ and $p \wedge \neg q$ are equivalent. (5 points)

p	q	p→q	¬(p→q)	¬q	p∧¬q
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

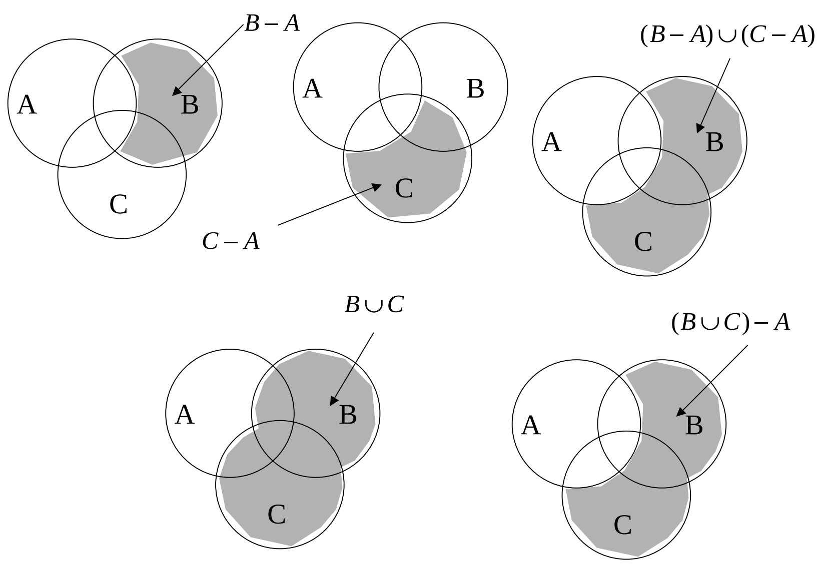
It can be observed the truth values of $\neg(p \to q)$ and $p \wedge \neg q$ are identical for all values of p and q.

(a) Use equivalence laws to show that $\neg(p \to q)$ and $p \wedge \neg q$ are equivalent. (5 points)

Using implication law and the De Morgan's law

$$\neg(p \to q) = \neg(\neg p \vee q)$$
$$= p \wedge \neg q$$

2. Use the Venn diagrams to show that $(C \cup B) - A = (C - A) \cup (B - A)$. (10 points)



3. Determine whether the following functions are bijections from **R** to **R**. Please explain.

(a) $f(x)=x^2$ (conclusion 1.5 points, explanation 3.5 points)

(a) $f(x)=x^2$ is not a bijection because it is not onto and not one-to-one
The function is not onto because there no function value will be negative. However, the co-domain include all real numbers
The function is not one-to-one because –x and x are mapped to the same point x^2

(b) $f(x) =x^3-2$ (conclusion 1.5 points, explanation 3 pints)

It is obvious that f(x) is a function, because the function give a single answer to each input.
The function is onto because for any real number y, we can find a $x=(y+2)^{1/3}$ so that f(x)=y
The function is one-to-one because, if f(x)=f(z), then $x^3-2= z^3-2$, then $x^3= z^3$, then x=y. So it is not possible for two different x and y, such that f(x)=f(z)
Since a function f(x) is both onto and one-to-one, it is a bijection

4. (a) Convert a decimal integer $(513)_{10}$ to its octal representation using the general base *b* expansion. (5 points)

Since

513=8*64+**1**
64=8*8+**0**
8=8*1+**0**
1=8*0+**1**

Therefore, $(513)_{10} =(1001)_8$

(b) Convert a Hexadecimal number $(5AD)_{16}$ to its decimal representation. (5 points)

$$5\cdot 16^2 + 10\cdot 16^1 + 13 = 1280 + 160 + 13 = (1453)_{10}$$

5. Use the Euclidean algorithms to find gcd(952, 340). (10 points)

952=340*2+272
340=272*1+68
272=68*4+0

Therefore, gcd(952, 340)=68

6. Compute the double summation $\sum_{i=0}^{100}\sum_{j=3}^{20}(2^i+3j)$. (10 points)

$$\sum_{i=0}^{100}\sum_{j=3}^{20}(2^i+3j)=\sum_{i=0}^{100}(18\cdot 2^i+3\sum_{j=3}^{20} j)=\sum_{i=0}^{100}(18\cdot 2^i+3\cdot 23\cdot 9)$$

$$=18\frac{2^{101}-1}{2-1}+101\cdot 3\cdot 23\cdot 9$$

7. Use mathematical induction to prove that $\sum_{i=1}^{n} \sqrt{i} > \frac{n}{2}$ for n ≥1. (10 points)

Let the statement P(n) means $\sum_{i=1}^{n} \sqrt{i} > \frac{n}{2}$

Basis Step: P(1) is true because $\sum_{i=1}^{1} \sqrt{i} = 1 > \frac{1}{2} = 1/2$

Inductive Step: We want to prove that P(n) → P(n+1), which is

If $\sum_{i=1}^{n} \sqrt{i} > \frac{n}{2}$ is true, then $\sum_{i=1}^{n+1} \sqrt{i} > \frac{(n+1)}{2}$ is true.

This can be proved if $\sqrt{n+1} > \frac{n+1}{2} - \frac{n}{2} = 1/2$. This is true for any n>1

8. Use mathematical induction to prove that 2 divides $(n^2 - 1)(n+2)$ for any positive integer n. (10 points)

Let the statement P(k) be $2|(k^2 - 1)(k+2)$

Basis step: P(1) is true because –0*3=0, which can be divided by 2

Inductive step: We want to prove that P(k) → P(k+1), which is

If $2 | (k^2 - 1)(k+2)$ then $2 | ((k+1)^2 - 1)(k+3)$

This can be proved by observing that

$$((k+1)^2 - 1)(k+3)$$
$$= k(k+2)(k+3)$$
$$= ((k-1)(k+1) + 3k + 1)(k+2)$$
$$= (k-1)(k+1)(k+2) + (3k+1)(k+2)$$
$$= (k^2 - 1)(k+2) + (3k+1)(k+2)$$

According to P(k), the first term is divisible by 2. For the second term, one of the factors must be even, so is even. As the result, P(k+1) is true.

9. Give a recursive definition of the sequence $\{a_n\}$, $n = 1,2,3,...$ if $a_n = \frac{n!}{n-1}$. (10 points)

$$a_n = \frac{n!}{n-1} = \frac{(n-1)!n(n-2)}{(n-2)(n-1)} = \frac{n(n-2)}{(n-1)} a_{n-1}$$

10. What is the probability that a five-card poker hand contains three queens, 1 jack, and one 5. (10 points)

C(4,3)C(4,1)C(4,1)/C(52,5)

11. (a) When rolling TWO dice, what is the probability of getting the sum 9. (10 points)

There are total of 6x6=36 outcomes. Sum 9 cases 3+6, 4+5, 5+4, 6+3, total of 4
So 4/36=1/9

(b) When rolling THREE dice, what is the probability of getting the sum 6. (bonus 10 points)

Total of 6x6x6=216 cases

1+1+4 there are 3 cases, 1,1,4; 1,4,1; 4,1,1;

1+2+3 there are 6 cases P(3,3)=6
2,2,2 only 1 case

total of 10 cases. So 10/216