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①

Quiz 2 Solutions

$$\textcircled{1} \quad x_{k+1} = x_k - x_k^2 + 1$$

(a) Compute fixed points x_*

i) plug in x_* for x_k, x_{k+1}

$$x_* = x_* - x_*^2 + 1$$

ii) solve for x_*

$$0 = \cancel{-x_*} + \cancel{x_*} - x_*^2 + 1$$

$$x_*^2 = 1 \Rightarrow \boxed{x_* = \pm 1}$$

~~$$x_{k+1} = x_k + \frac{1}{8} x_k^3 + 1$$~~

$$x_{k+1} = x_k - \frac{1}{8} x_k^3 + 1$$

i) ~~$x_* = x_* - \frac{1}{8} x_*^3 + 1$~~

ii) $\frac{1}{8} x_*^3 = 1 \Rightarrow x_*^3 = 8$

$x_* = \sqrt[3]{8} = 2$

b) $x_1 = x_* + 0.1$ ~~$x_* = 2$~~
 $(x_1, x_2, x_3, x_4, x_5, x_6)$

$x_* = 2$

$x_1 = 2.1$

$(2.1, .89, 1.09, 0.901, 1.09, 0.903)$

$x_2 = f(x_1) = x_1 - x_1^2 + 1$

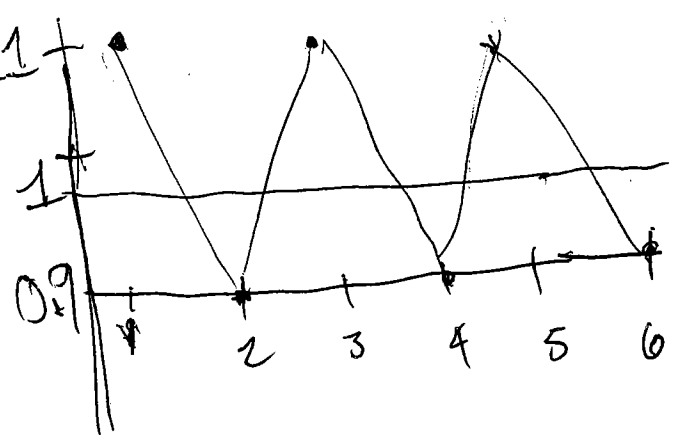
$x_2 = 2.1 - 2.1^2 + 1 = 2.1 - 4.41 + 1 = -1.31$
 $= .89$

$x_3 = x_2 - x_2^2 + 1 = 1.09$

$x_4 = 0.901$

$x_5 = 1.089$

$x_6 = 0.903$



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Quiz 2 cont'd

3. No fixed points

$$x_{k+1} = x_k + 1$$

$$\rightarrow \cancel{x_*} = \cancel{x_*} + 1$$

$$0 = 1 \quad \cancel{x}$$

4. $x_{k+1} = ?$ (Infinitely many)
 $\quad \quad \quad = \underline{x_k}$

3 $\xrightarrow{\hspace{10em}}$

$$\boxed{x_{k+1} = x_k - \sin(x_k)}$$

$$x_* = 0, \pm\pi, \pm 2\pi, \dots$$

$$\cancel{x_*} = \cancel{x_*} - \underbrace{\sin(x_*)}_{=0} = 0$$

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②

Example of computing an orbit near a fixed point

$$X_{k+1} = X_k - \sin(X_k)$$

$$X_* = 0$$

Start near $X_* = 0$, at $x_1 = 0.1$ (radians)

Compute the orbit $(x_1, x_2, x_3, x_4, x_5)$.

set $k=1$.

$$x_2 = x_1 - \sin(x_1) = 0.1 - \sin(0.1) \\ = \underline{1.7 \times 10^{-4}}$$

$$x_3 = x_2 - \sin(x_2) = 7.7 \times 10^{-13}$$

$$\underline{(0.1, 0.00017, 10^{-13}, 0, 0)}$$

$X_* = \pi$, starting at $x_1 = \pi + 0.1 \approx 3.24$
generate orbit: $(x_1, x_2, x_3, x_4, x_5)$

(3.24, 3.34, 3.51, 3.93, 4.64)

A sequence of numbers is shown in parentheses: (3.24, 3.34, 3.51, 3.93, 4.64). Below the numbers, four curved arrows point from each number to the next one to its right. Under each arrow is a handwritten value representing the difference between the two numbers it connects: .10, ~~.17~~, .42, and .71.

From	To	Difference
3.24	3.34	.10
3.34	3.51	.17
3.51	3.93	.42
3.93	4.64	.71