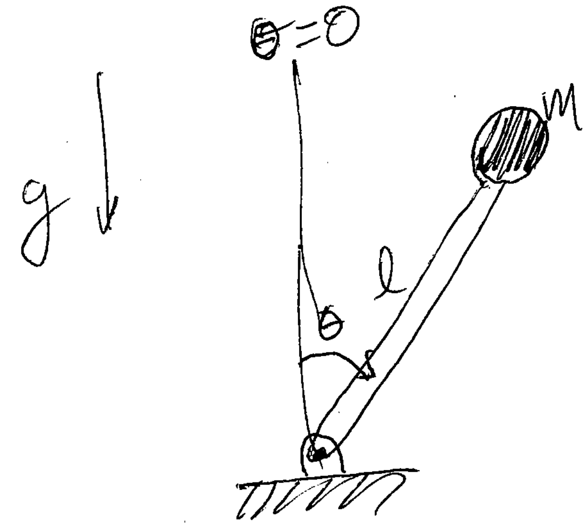
## pendulum




objective is
① $\theta^* = 0$ to be eq. pnt.
② for $\theta^* = 0$ to be stable & attractive eq. pnt.

① $x^* = \cos(x^*)$
(equilibrium point $x^*$)

② for any starting value $x_1$,

$$x_n \xrightarrow{n \to \text{big enough}} x^*$$

(for n big enough, $x_n = x^*$)
In this case, $x^*$ is a stable and attractive eq. pnt.

10/16/07



$$x_{n+1} = f(x_n)$$

$$f(x) = e^{-x}$$

$$x_{n+1} = e^{-x_n}$$

---

$$x_{n+1} = \cos(x_n)$$

---

Fixed point $x_*$ satisfies

$$x_* = f(x_*)$$

The <u>orbit</u> of a model is just the vector

$(x_1, x_2, \ldots, x_N)$ given $x_1, N$

$\rightarrow (x_1, f(x_1), f(f(x_1)), \ldots)$

10/18/07 ①

model $x_{k+1} = f(x_k)$

orbit (given $x_1$, $N$) is

$(x_1, x_2, \ldots, x_N)$

equilibrium is any point(s) $x_*$ satisfying $x_* = f(x_*)$

---

Task 3.3.2

$$x_{k+1} = x_k + x_k^2 - 1$$

To determine equilibrium, simply plug in $x_*$ for $x_k$ and $x_{k+1}$, and solve for $x_*$

$$x_* = x_* + x_*^2 - 1$$
$$-x_* \qquad -x_*$$

10/18/07 

$$0 = x_*^2 - 1$$

$$+1 \qquad +1$$

$$1 = x_*^2$$

ans. $x_* = \pm\sqrt{1} = \pm 1$ //

$$x_{k+1} = x_k + x_k^2 - 1$$

orbit starting from $x_1 = -1$, $N = 5$

is $(-1, -1, -1, -1, -1)$

10/18

## Models with No equilibrium points:

1) $x_{k+1} = x_k + 1 \to c$

$x_* = x_* + c$ never true $c \neq 0$

$0 = 1$ ✗

2) $x_{k+1} = x_k^2 + 1$

$x_* = x_*^2 + 1$

$0 = x_*^2 - x_* + 1$

$x_* = 1 \pm \sqrt{1-4}$

$\sqrt{-3}$

## Models with infinitely many equilib.

1) $x_{k+1} = x_k$

$\theta_{k+1} = \theta_k + \Delta \cdot v$, $v = 0$

speed of car

$v_{k+1} = v_k + \left(\frac{\Delta}{m}\right)\left[-b v_k + h_k\right]$

$v_k$ - speed at time $t_k$

$\Delta$ - sample period

$m$ - mass

$b$ - friction

$h_k$ - hill incline/decline

10/28 (4)

flat $h_k = 0$

inclined $h_k < 0$

declined $h_k > 0$

$b > 0$

$$V_* = V_* + \left(\frac{\Delta}{m}\right)\left[-bV_* + h_k\right]$$

$\left(\frac{\Delta}{m}\right)$ : constant $\neq 0$

$\left[-bV_* + h_k\right] = 0$

$$\text{Const} * \left[\quad\right] = 0$$

$\left[\quad\right] = 0$

$$-bV_* + h_k = 0$$

ans. $V_* = \frac{h_k}{b}$

flat: $h_k = 0$

$V_* = 0$

incline: $h_k < 0$

$V_* = \frac{h_k}{b} < 0$

decline: $h_k > 0$

$V_* = \frac{h_k}{b} > 0$