

AMS 206 – Homework Assignment #8  
due Tuesday, March 20, 10am at BE 151

Required Problem

1. The data file `printlog.txt` contains three weeks worth of data on the number of pages printed by each person in the school of engineering, so that each row represents a person and each column gives the number of pages that person printed that week. Denote the number of pages printed in week  $k$  by person  $j$  as  $y_{jk}$ . We model the number of pages printed in a week as Poisson, but we figure that some people are heavier users of the printer than others, so we want to fit a hierarchical model with a separate mean  $\lambda_j$  for each user. Put a  $\text{Gamma}(\alpha, 1)$  prior on the  $\lambda_j$ 's. Use a somewhat vague prior for  $\alpha$ , an exponential with mean 100. Thus the hierarchical model is

$$\begin{aligned} Y_{jk} | \lambda &\stackrel{iid}{\sim} \text{Pois}(\lambda_j) && \text{for } k = 1, 2, 3 \\ \lambda_j | \alpha &\stackrel{iid}{\sim} \text{Gamma}(\alpha, 1) \\ \alpha &\sim \text{Exp}\left(\frac{1}{100}\right) \end{aligned}$$

- (a) Find the complete conditional distribution for a  $\lambda_j$  and recognize it as something that can be sampled with a Gibbs step.
- (b) Find the complete conditional distribution for  $\alpha$ .
- (c) Fit the model with MCMC. (Don't forget to do the necessary tuning. Also note that the  $\lambda$  matrix will have different dimensions from the class example.) Show your trace plots for  $\alpha$  and at least three  $\lambda_j$ 's of your choice.
- (d) Make a plot comparing the maximum likelihood estimates of the  $\lambda_j$ 's to your estimated posterior means of the  $\lambda_j$ . Use `abline(0, 1)` to add the  $y = x$  line to your plot. Comment on what you see. How does this Bayesian analysis compare to a simple frequentist (maximum likelihood) one?

(For the record, this is an oversimplification of the model. For example, the Poisson likelihood isn't that good of a fit because the week-to-week variation is larger than we'd expect from Poisson data. But we'll just keep things simple for this homework assignment.)

Optional Problem

2. Expand the above model so that the prior for each  $\lambda_j$  is a  $\text{Gamma}(\alpha, \beta)$  prior, and put somewhat vague exponential priors on  $\alpha$  and  $\beta$  with means 100 and 10 respectively, i.e.,

$$\begin{aligned} Y_{jk} | \lambda &\stackrel{iid}{\sim} \text{Pois}(\lambda_j) && \text{for } k = 1, 2, 3 \\ \lambda_j | \alpha, \beta &\stackrel{iid}{\sim} \text{Gamma}(\alpha, \beta) \\ \alpha &\sim \text{Exp}\left(\frac{1}{100}\right) \\ \beta &\sim \text{Exp}\left(\frac{1}{10}\right) \end{aligned}$$

Fit this model (note that  $\beta$  can be fit with Gibbs steps) and compare the results to the above model and to the MLE's.