

AMS 206 – Homework Assignment #7
due Friday, March 9

Required Problems

1. For the `cereal` data, let's take a look at the grams of carbohydrates (carbo) per serving. Model each cereal's carb count as independent Poissons with unknown mean λ . Before looking at the data, you might not know much about how many carbs to expect in a serving, so let's use a random prior, a Gamma prior with parameters 5 and β , where β is also unknown. Let β have a prior which is exponential with mean 1/2. Thus:

$$\begin{aligned}y_i &\stackrel{iid}{\sim} Pois(\lambda) \\ \lambda &\sim \Gamma(5, \beta) \\ \beta &\sim Exp(2)\end{aligned}$$

- (a) Read the data into R. Make a histogram of the carbohydrate data. If you notice any problems, fix them and re-do the histogram. Is the Poisson assumption reasonable?
 - (b) Find the complete conditional distributions for λ and β .
 - (c) Fit the model using MCMC. You should be able to use Gibbs sampling for both parameters.
 - (d) After discarding burn-in if necessary, make trace plots and histograms for both parameters. Do the shapes of the histograms make sense? What are the posterior mean and standard deviation of each of the parameters?
2. Consider again the problem of counting deer on campus, i.e., a binomial likelihood with both ν and θ unknown, and only a single observation Y is available. Show that bad things happen if you try to put a uniform prior on each of ν and θ , i.e., $f(\nu) \propto 1$ for all counting numbers (positive whole numbers), and $f(\theta) = I_{\{0 \leq \theta \leq 1\}}$. In particular:
 - (a) Find the joint posterior $f(\nu, \theta | \mathbf{y})$. Next find the marginal posterior for ν by integrating out θ , i.e., $f(\nu | \mathbf{y}) \propto \int_0^1 f(\nu, \theta | \mathbf{y}) d\theta$. (You should be able to recognize that integral as a familiar pdf.)
 - (b) What happens when you try to evaluate the sum $\sum_{\nu=0}^{\infty} f(\nu | \mathbf{y})$? With the right normalizing constant do you get 1?
 - (c) Using noninformative priors often gives similar results to frequentist methods. Do you get a "similar" result in this case?
 - (d) Suppose you hadn't done the math above, and you just blindly tried to fit that model using MCMC. Use the datum $Y = 18$ and fit the model with MCMC. Note θ can be sampled with a Gibbs step, while ν requires a Metropolis-Hastings step (make sure to propose only whole numbers – one easy way to do this is to propose draws from a normal or uniform then use the `round` function). Use enough iterations, say at least 10,000. What happens? Is this consistent with your answer from the previous parts?

Optional Problem

3. In class, Gibbs sampling for normal data with unknown mean and variance was presented, but the complete conditional for τ required the evaluation of $\sum_i (y_i - \mu_{t+1})^2$ at each iteration, which could be a lot of work for a larger dataset. Re-write the complete conditional for τ so that it only depends on hyperparameters and the sufficient statistics \bar{y} and $\sum_i (y_i - \bar{y})^2$, which can be computed once before starting MCMC, rather than re-computed at each iteration. Run a Gibbs sampler for the alcohol content in the `beer` data using this complete conditional.