

AMS 206 – Homework Assignment #2  
due Wednesday, January 24

Required Problems

1. One of the things that most bothers me about the Windows operating system is when it crashes when I'm trying to shut it down. It seems to be more likely to crash when I have put the computer on standby (sleep) earlier in the day. Suppose the probability that it crashes when it has been on standby earlier is 0.1, and the probability it crashes when it has not been on standby is 0.01, and that the probability that I put it on standby during a day's usage is 0.4. If you just walked into the room in time to see me trying to shutdown and have the computer crash, what is the probability that I had put it on standby earlier that day?
2. (2.1.a) Consider three prisoners A, B, and C, exactly one of whom will be pardoned; the other two will be executed. Let A, B, and C, respectively denote the events that prisoner A, B, or C will be pardoned. In the absence of any other evidence to the contrary, it is reasonable to assume the "prior probabilities"  $p(A) = p(B) = p(C) = \frac{1}{3}$ . The warden now enters prisoner A's cell and tells him that B will be executed. Assume that the warden will always truthfully tell A which one of B or C is being executed (if the other one is being pardoned), and that if both are being executed, he will pick one with equal probability to tell A.
  - (a) Are these assumptions reasonable?
  - (b) Use Bayes' Theorem to show that the warden did not provide A with any useful information, i.e., the probability of A being pardoned conditioned on the information is unchanged.
  - (c) Prisoner C is listening through a hole in the wall. What is his updated probability of execution (note: the book is asking for the probability of execution, not probability of pardon).
3. (2.1.d, modified) A robbery at Harrods in London is observed to have been committed by a Royal Marine.
  - (a) You are informed that a population of  $N=20,000$  Royal Marines had access to Harrods at the time of the robbery. Without any further information, what is your only fair prior probability  $\phi$  that any particular marine is the robber?
  - (b) Evidence is then produced that the robber had a Union Jack tattooed on his forehead (this feature will be referred to as the I.D. evidence). You are informed that exactly  $M=20$  out of the  $N=20,000$  marines have this emblem. In the absence of further information, what is your posterior probability  $\phi^*$  that any marine out of these 20 is the robber?
  - (c) The computer records of the  $N$  marines are then randomly sampled until a marine is discovered with the I.D. evidence. He is immediately arrested and charged with the robbery. No further evidence is submitted. The likelihood ratio  $R^*$  is reported, where

$$R^* = \frac{p(\text{I.D. evidence}|\text{defendant})}{p(\text{I.D. evidence}|\text{random marine})} = \frac{1}{M/N} = 1000$$

The prosecuting counsel judges that the odds are 1000 to 1 against the defendant when compared with a random man from the population. Is this fair? Explain.

- (d) A restatement of Bayes' Theorem, known as the Essen-Möller formula, tells us that the suspect's posterior probability of guilt is

$$\lambda^* = \frac{R\lambda}{R\lambda + 1 - \lambda},$$

where  $R$  is a separately defined likelihood ratio, and  $\lambda$  is the prior probability of guilt. A re-arrangement of terms gives the equivalent formula

$$\frac{\lambda^*}{1 - \lambda^*} = \frac{R\lambda}{1 - \lambda}.$$

Show that in the Royal Marine case, we must have  $R = \frac{N-1}{M-1}$  (plug in values for  $\lambda$  and  $\lambda^*$  and solve for  $R$ ; note that there is a typo in the book). Interpret this result.

4. Recall the artichoke data from HW #1 (and the class example). You might complain that we are counting whole numbers of artichokes, so we should model the data using a discrete distribution. Suppose we consider only Poisson and Geometric distributions, each with mean 27, and consider them with equal prior probability. Given the seven observations, what is the posterior probability that they are from a Poisson distribution? Is this reasonable? (Consider the spread of the data and the theoretical variances of the proposed distributions. Also, does this consideration improve the appeal of using a continuous distributions with two parameters?)

#### Optional Problem

5. (2.1.e) Consider the Essen-Möller formula (above) for general  $\lambda$ , when

$$R = \frac{1}{p(\text{I.D. evidence}|\text{random marine})},$$

but where the random marine is chosen from the  $N - 1$  members of the population excluding the defendant. Hence  $R^{-1}$  averages the  $N - 1$  probabilities  $\phi_i = p(\text{I.D. evidence}|i\text{th member})$  for  $i \in \{2, 3, \dots, N\}$ , for the  $N - 1$  individuals.

- (a) Show that the Essen-Möller formula is then equivalent to a version of Bayes' Theorem which assigns prior probability  $\lambda$  to the defendant and prior probabilities  $\frac{1-\lambda}{N-1}$  to each of the remaining  $N - 1$  members of the population (i.e., work out Bayes' Theorem and re-arrange the result to show that you get the Essen-Möller formula with this  $R$ ).
- (b) Would you regard the choice of  $\lambda = \frac{1}{2}$  as neutral? (The only two options are that the defendant is guilty or not guilty, so  $\lambda = \frac{1}{2}$  puts equal probability on these two outcomes.) Would a choice of  $\lambda$  exceeding  $N^{-1}$  indicate the present of prior evidence against the defendant?