AMS 206 – Homework Assignment #4
due Wednesday, February 8

Required Problems

1. Suppose you are consulting for a manufacturer of tea bags. Each bag is supposed to contain 5.5 grams of tea, and the filling process is known to have a standard deviation of 0.106 grams per bag. Of interest is whether there is a drift (change) in the mean amount of tea per bag.

   (a) Determine a prior distribution that has a mean matching the process specification and contains the information of about one hundred observations.

   (b) A sample of 40 bags finds an average of 5.494 grams per bag.

      i. What is the posterior probability that the process is within 0.001 grams of its specification? (Be sure to keep enough decimal places in your computations.)

      ii. What is the posterior predictive probability that a new tea bag will contain at least the labeled amount of 5.5 grams of tea?

2. The density for the Pareto distribution is given in Equation (2.132) on p. 119.

   (a) Show that the inverse CDF for the Pareto is $F^{-1}(p) = \beta(1-p)^{-1/\alpha}$.

   (b) Find the mean and variance of a Pareto distribution with parameters $\alpha = 4$ and $\beta = 5$ by Monte Carlo estimation (generate samples using the inverse CDF method), and compare them to the theoretical values given on p. 119.

   (c) Define a new distribution by $Y = \log(X - \beta)$ where X has a Pareto(α,β) distribution (and log is the natural log). Estimate the mean and variance of this distribution for $\alpha = 4$ and $\beta = 5$ (transform your sample from the previous part).

Optional Problem

3. Show that for a normal likelihood with unknown mean and unknown precision (or variance), the marginal posterior distribution for the mean is a scaled t. The density for the scaled t is given in Equation (2.102) on page 101. You will probably want to use the form for the joint posterior that has the square already completed for μ.