AMS 206 – Homework Assignment #1
due Wednesday, January 18

Required Problems

1. **Do the parts of this question in order, and do each part before reading the next part!** Consider the four events A_1, A_2, A_3, and A_4, which are described as follows:
   A_1 is the event that the area of the state of Pennsylvania is less than 5,000 sq. miles
   A_2 is the event that the area of the state of Pennsylvania is between 5,000 and 50,000 sq. miles
   A_3 is the event that the area of the state of Pennsylvania is between 50,000 and 100,000 sq. miles
   A_4 is the event that the area of the state of Pennsylvania is more than 100,000 sq. miles

   (a) Without using any outside information, assign your subjective probabilities to these four events. Do this before reading the rest of the question!

   (b) You are now given the information that the area of Alaska (the largest of the 50 states) is 586,000 sq. miles and the area of Rhode Island (the smallest) is 1,214 sq. miles. In light of this information, determine your subjective probablities of the four events.

   (c) You are now given the information that when area is considered, Pennsylvania is the 33rd largest of the 50 states (i.e., 32 states are larger). Update your probabilities.

   (d) You are now given the information that the area of New York, the 30th largest state, is 49,576 sq. miles. Update your probabilities.

2. One of the things that most bothers me about the Windows operating system is when it crashes when I'm trying to shut it down. It seems to be more likely to crash when I have put the computer on standby (sleep) earlier in the day. Suppose the probability that it crashes when it has been on standby earlier is 0.1, and the probability it crashes when it has not been on standby is 0.01, and that the probability that I put it on standby during a day's usage is 0.4. If you just walked into the room in time to see me trying to shutdown and have the computer crash, what is the probability that I had put it on standby earlier that day?

3. Do problem 1.4.3 (p. 14) from the text.

4. Suppose a person has \$A ($A > 0$) and can bet any amount b of this fortune ($0 \leq b \leq A$) on a game where they win \$b with probability p and lose \$b with probability $1-p$ (i.e., their fortune changes to $A + b$ w.p. p, and changes to $A - b$ w.p. $1-p$). Suppose this person's utility function is logarithmic: $U(x) = log(x)$.

   (a) Using the principle of maximum expected utility, determine the optimal value of b (as a function of A and p).

   (b) Interpret your results.

5. At night, the perimeter shuttle runs every 20 minutes. Suppose the waiting time for the next shuttle is exponentially distributed. Find the probability that you have to wait between 10 and 15 minutes (do this by evaluating the integral of the pdf).

6. Let $X_1, \ldots, X_n \overset{iid}{\sim} Pois(\lambda)$ with $\lambda > 0$ unknown.

   (a) Find the maximum likelihood estimator of λ asuming that at least one of the observations X_i is larger than zero.

   (b) Show that the MLE does not exist if all the observations are zero.

Optional Problem

7. Consider the case where you observe data $X_1, \ldots, X_n$ which are modeled as $iid\ N(\mu, \frac{1}{\tau})$. Note that using the precision (instead of the variance) will make the computations easier.

   (a) Write out the likelihood function for μ and τ, $L(\mu, \tau|\mathbf{x}) = f(\mathbf{x}|\mu, \tau)$.

   (b) Find the marginal likelihood for τ, i.e., $L(\tau|\mathbf{x}) = f(\mathbf{x}|\tau) = \int f(\mathbf{x}|\mu, \tau)d\mu$, the likelihood function for τ when μ has been integrated out. You will need to expand the square in the exponential term and then complete the square for μ. After you have integrated out μ, simplify the resulting expression for τ to see that it has the form of a gamma distribution (but without the right normalizing constant).

   (c) Find the maximum likelihood estimate for τ using (i) the original likelihood (with τ and μ) and (ii) the marginal likelihood.

   (d) The two MLEs should be very similar, but not quite the same. Explain how and why they differ in this case.