

ENG-27: Homework 3

1. Find the value of k for which the system,

$$\begin{aligned}x + 5y + 3z &= 0 \\5x + y - kz &= 0, \\x + 2y + kz &= 0\end{aligned}$$

has a nontrivial solution. For this special value of k , find the solution.

2. Using an elimination scheme, solve the equations

$$\begin{array}{ll}x_1 + x_2 = 3 & -x_1 + 2x_2 + x_3 = 2 \\(i) \quad 2x_1 + x_2 + x_3 = 7 & (ii) \quad x_2 - 2x_3 = -3 \\x_1 + 2x_2 + 3x_3 = 14 & x_1 + 4x_2 - x_3 = 4\end{array}$$

3. For what values of k does the system,

$$\begin{aligned}x_1 + x_2 + x_3 &= 1 \\x_1 + 2x_2 + 4x_3 &= k, \\x_1 + 4x_2 + 10x_3 &= k^2\end{aligned}$$

have a solution? What are the solutions for these special values of k ?

4. Using the Gauss-Jordan method (i.e. row operations on an augmented matrix), invert the matrices,

$$(i) \quad A = \begin{pmatrix} -1 & 2 & 1 \\ 0 & 1 & -2 \\ 1 & 4 & -1 \end{pmatrix}, \quad (ii) \quad A = \begin{pmatrix} 5 & 2 & 4 \\ 3 & -1 & 2 \\ 1 & 4 & -3 \end{pmatrix},$$

5. The location of a particle in a fluid can be represented by the vector $(x_1, x_2, x_3)^T$. *N.B. Transpose here simply means that the row vector becomes a column vector.* Under a certain fluid motion, it is discovered that the particle $x = (x_1, x_2, x_3)^T$ moves to the new position Ax , where

$$A = \frac{1}{9} \begin{pmatrix} 8 & -1 & -4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{pmatrix}.$$

(a) Where does the particle at $x = (2, 1, 1)^T$ move to? (b) If the new position is $(2, 1, 1)^T$, where did the particle originate? (c) Find the positions of particles that do not move in the fluid motion.

6. Find the characteristic polynomial, eigenvalues and eigenvectors of the matrices:

$$(i) \quad A = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}, \quad (ii) \quad A = \begin{pmatrix} 1 & 1 & 3 \\ 0 & 2 & 1 \\ 4 & -4 & -1 \end{pmatrix},$$

7. Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 3 & 3 & 2 \\ 2 & 4 & 2 \\ -1 & -3 & 0 \end{pmatrix}.$$

Take the eigenvectors and create a new matrix,

$$P = \begin{pmatrix} x_1^a & x_1^b & x_1^c \\ x_2^a & x_2^b & x_2^c \\ x_3^a & x_3^b & x_3^c \end{pmatrix},$$

where $x^a = (x_1^a, x_2^a, x_3^a)^T$, $x^b = (x_1^b, x_2^b, x_3^b)^T$ and $x^c = (x_1^c, x_2^c, x_3^c)^T$ are the three eigenvectors (ignore any arbitrary constants multiplying the eigenvectors). Construct P^{-1} , and then $P^{-1}AP$. What is special about this last matrix? (This procedure is called the "diagonalization" of a matrix.)