ENG-27: Homework 1

1. Find the general solutions of the differential equations,

$$(a)\quad y' = xy^2 \qquad (b)\quad y' + e^{2x}y^2 = 0$$

$$(c)\quad y' + 3y\sin\omega x = 0 \qquad (d)\quad y' = \frac{2}{x}\sqrt{1-y^2},$$

where ω is a constant.

2. Find the general solutions of the differential equations,

$$(a)\quad y' - \frac{2}{x}y = x^2e^x \qquad (b)\quad y' + 2xy = x$$

$$(c)\quad xy' + y = x + x^3 \qquad (d)\quad (1-x^2)y' + xy = x.$$

3. Solve the initial-value problems,

$$(a)\quad y' + y = (x+1)^2, \quad y(0) = 0 \qquad (b)\quad x' = \frac{x^2}{t+1}, \quad x(0) = 1$$

$$(c)\quad tx' + x = t^2, \quad x(2) = 1/3 \qquad (d)\quad x' = e^{x+t}, \quad x(0) = a,$$

with a a constant.

4. Find the general solutions of the differential equations,

$$(a)\quad y'' - y = 0 \qquad (b)\quad y'' + y = 0$$

$$(c)\quad y'' + 8y' + 15y = 0 \qquad (d)\quad y'' + 6y' + 13y = 0$$

$$(e)\quad y'' - 3y = 0 \qquad (f)\quad y''' - y'' - 4y' + 4y = 0$$

$$(g)\quad y''' - y'' + 4y' - 4y = 0 \qquad (h)\quad y'' - 6y' + 9y = 0.$$

5. Find the general solutions of the differential equations,

$$(a)\quad y'' + 4y' - 12y = 14e^x \qquad (b)\quad y'' - 13y' + 12y = 36x$$

$$(c)\quad y'' + 4y' + 3y = 8\cos x - 6\sin x$$

6. Find the solutions of the differential equations,

$$(a)\quad y'' + 4y' - 12y = 14e^x, \qquad y(0) = 1, \quad y'(0) = -4$$

$$(b)\quad y'' + 4y' - 12y = 14e^{2x}, \qquad y(0) = 0, \quad y'(0) = 0$$

$$(c)\quad y'' + 4y' - 12y = 3, \qquad y(0) = 1, \quad y \text{ finite as } x \to \infty.$$

7. Find the solution to:

$$y' + y - z = e^t \qquad z' + z - y = e^t$$