ENGR 7 W03 HW4 solutions

(total 100 pts)

1. Problem 11 pg. 255 [10 pts]

(a) Construct a 95% confidence interval for the population mean μ 3pts

Use the t-distribution with 58-1 = 57 degrees of freedom

95% of values will fall between t=-2.002 and t=2.002

A two sided confidence interval is then

$(25 - 2\frac{2.7}{\sqrt{58}}, 25 + 2\frac{2.7}{\sqrt{58}})$

$= (24.3, 25.7)$

(b) What is the p-value of the test 3pts

$H_o : \mu = 24$

$H_a : \mu \neq 24$

$t = \frac{\bar{x} - \mu_o}{\frac{s}{\sqrt{n}}}$

$= \frac{25-24}{\frac{2.7}{\sqrt{58}}}$

$= 2.82$

So $2(0.0005) < p < 2(0.005)$

$= 0.001 < p < 0.01$

So we reject H_o

(c) What do you conclude? 2 pts

The mean baseline body mass index for the population of men who later develop diabetes is not equal to 24, the mean for the population of men who do not. It is higher.

(d) Based on the 95% confidence interval, would you have expected to reject or not to reject the null hypothesis? 2pts

The value 24 does not lie inside the 95% confidence interval for μ so we would expect to reject the null hypothesis.

2. Problem 5, page 278 [20 pts]

(a) are the two samples of data paired or independent? 2pts

paired

(b) What are the null and alternative hypothesis for a 2-sided test? 2pts

$H_o : \mu_{corn} - \mu_{oats} = 0$

$H_a : \mu_{corn} - \mu_{oats} \neq 0$

(c) Conduct the test at the 0.05 level of significance. p-value?

first, calculate the difference in LDL for each subject

Subject	*Difference*
1	0.77
2	0.85
3	−0.45
4	−0.26
5	0.30
6	0.86
7	0.60
8	0.62
9	0.31
10	0.72
11	0.09
12	0.16
13	0.41
14	0.10

$\bar{d} = 0.363(5pts)$

$s_d = 0.406(5pts)$

$t = \frac{\bar{d}-\delta}{\frac{s_d}{\sqrt{n}}}$

$= \frac{0.363-0}{\frac{0.406}{\sqrt{14}}}$

= 3.35 (4 pts)

df = 14 - 1 = 13

therefore, $0.001 < p < 0.01$

we reject H_o at 0.05 level of significance (2pts)

(d) What do you conclude?

The true difference in population mean cholesterol levels is not equal to 0. Mean LDL cholesterol is lower when individuals use the oat bran diet.

3. Problem 7, page 279 [10 pts]

(a) Construct a 1-sided 95% confidence interval for the true difference in population means $\mu_{12} - \mu_{24}$

Subject	*Difference*
1	49
2	31
3	18
4	34
5	33
6	7
7	104

$\bar{d} = 39.4(2pts)$

$s_d = 31.4(2pts)$

Using a t-distribution with 7 - 1 = 6 df

95% of the values lie above -1.943

$\bar{\delta} \geq \overline{d} - 1.943(\frac{s_d}{\sqrt{n}})$

$= 39.4 - 1.943(\frac{31.4}{\sqrt{7}})$

$= 16.3$ (2 pts)

(b) Test the null hypothesis that the population mean are identical at the $\alpha = 0.05$ **level of significance. What do you conclude? [5 pts]**

$H_o : \mu_{12} - \mu 24 \leq 0$

$H_a : \mu_{12} - \mu_{24} > 0$

$\delta = \mu_{12} - \mu_{24} = 0$

$t = \frac{\overline{d} - \delta}{\frac{s_d}{\sqrt{n}}}$

$= \frac{39.4-0}{\frac{31.4}{\sqrt{7}}}$

$= 3.32$

Therefore, $0.005 < p < 0.01$ and we reject H_o at 0.05 level of significance. We conclude that the true difference in population mean cotinine levels is not equal to 0. Mean cotinine level decreases significantly between 12 and 24 hours after smoking.

4. Problem 8, page 280 [10 pts]

(a) Are the two samples of data paired or independent (2 pts)

Independent

(b) State the null and alternative hypotheses of the 2-sided test (2 pts)

$H_o : \mu_1 = \mu_2$

$H_a : \mu_1 \neq \mu_2$

(c) Conduct the test at the 0.05 level of significance.

What do you conclude?

$s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$

$= \frac{(77-1)(0.026)^2+(161-1)(0.025)^2}{77+161-2}$

$= 0.00064$ (3 pts)

$t = \frac{(\bar{x_1}-\bar{x_2})-(\mu_1-\mu_2)}{\sqrt{s_p^2(\frac{1}{n_1}+\frac{1}{n_2})}}$

$= \frac{(0.098-0.095)-0}{\sqrt{0.00064(\frac{1}{77}+\frac{1}{161})}}$

$= 0.86$ (3 pts)

So with 77 + 161 - 2 = 236 df, the t-distribution can be approximated with the normal distribution. Therefore p = 2(0.195) = 0.390, and we are unable to reject H_o.

The study fails to provide evidence that maternal cigarette smoking has an effect on the bone mineral content of newborns.

5. Problem 9, page 280 [10 pts]

(a) At the 0.01 level of significance, test the null hypothesis that the two populations of women have the same mean arterial blood pressure. (5 pts)

$H_o : \mu_1 = \mu_2$

$H_a : \mu_1 \neq \mu_2$

Since $s_1 = s_2 = 8$

$s_p^2 = 8^2 = 64$

$t = \frac{(\bar{x_1} - \bar{x_2}) - (\mu_1 - \mu_2)}{\sqrt{s_p^2(\frac{1}{n_1} + \frac{1}{n_2})}}$
$= \frac{(111-109)-0}{\sqrt{64(\frac{1}{23}+\frac{1}{24})}}$
$= 0.86$

for a t-distribution with 23 + 24 - 2 = 45 df, $p > 0.1$. therefore, we cannot reject H_o at the 0.01 level of significance. We do not have any evidence that mean arterial blood pressure differs for the two population of women.

(b) Construct a 95% confidence interval for the true difference in population means. does this interval contain the value 0? (5 pts)

Approximate 45 df with 40 df

99% of the values are contained within -2.704 and 2.704

$(\bar{x_1} - \bar{x_2}) \pm 2.704\sqrt{s_p^2(\frac{1}{n_1} + \frac{1}{n_2})}$
$= (111 - 109) \pm 2.704\sqrt{64(\frac{1}{23} + \frac{1}{24})}$
$= (-4.3, 8.3)$

This interval contains the value 0, and this is what we expect since we failed to reject the null hypothesis.

6. Problem 11, page 280 [20 pts]

(a) What are the null and alternative hypothesis of the 1-sided test? (5 pts)

$H_o : \mu_1 \geq \mu_2$

$H_a : \mu_1 < \mu_2$

(b) test at 0.05 level of significance and conclusion.

use two-sample t-test

$t = \frac{(\bar{x_1} - \bar{x_2}) - (\mu 1 - \mu 2)}{\sqrt{(\frac{s_1^2}{n_1}) + (\frac{s_2^2}{n_2})}}$

= -10.79 (5 pts)

$$\nu = \frac{[(\frac{s_1^2}{n_1})+(\frac{s_2^2}{n_2})]^2}{[\frac{(\frac{s_1^2}{n_1})^2}{(n_1-1)}+\frac{(\frac{s_2^2}{n_2})^2}{n_2-1)}]}$$

$$= \frac{[(\frac{1.69}{121})+(\frac{4}{75})]^2}{[\frac{(\frac{1.69}{121})^2}{(121-1)}+\frac{(\frac{4}{75})^2}{(75-1)}]}$$

= 113.1 (5 pts)

so using $\nu = 113df, p < 0.0005$. therefore, reject H_o at the 0.05

level of significance and conclude that the mean carboxyhemoglobin level

of the nonsmokers is lower than the mean level of the smokers (5 pts)

Problem 7 [20 pts]

(5 pts)

$H_o : \mu_1 - \mu_2 = 0$

$H_a : \mu_1 - \mu_2 < 0$

$$t = \frac{(\bar{x_1}-\bar{x_2})-(\mu 1-\mu 2)}{\sqrt{(\frac{s_1^2}{n_1})+(\frac{s_2^2}{n_2})}}$$

$$t = \frac{(972.1-843.4)-0}{\sqrt{(\frac{245.1^2}{12})+(\frac{251.2^2}{15})}}$$

= 1.341 (5 pts)

$$\nu = \frac{[(\frac{s_1^2}{n_1})+(\frac{s_2^2}{n_2})]^2}{[\frac{(\frac{s_1^2}{n_1})^2}{(n_1-1)}+\frac{(\frac{s_2^2}{n_2})^2}{n_2-1)}]}$$

$$= \frac{[(\frac{245.1^2}{12})+(\frac{251.2^2}{15})]^2}{[\frac{(\frac{245.1^2}{12})^2}{(12-1)}+\frac{(\frac{251.2^2}{15})^2}{(15-1)}]}$$

= 23.96 (5 pts)

Looking at df = 23, 0.05 < upper-tail area < 0.1, since in this case p-value =1 - upper-tail area, so 0.9 < p-value < 0.95, thus we would not reject the null hypothesis at either the 0.05 level of significance or the 0.1 level of significance. (5 pts)